CLAREZA ACADEMY

Grade 8 -- Mathematics Math

Free sample: Unit 1, Lessons 1 to 9

These are 9 lessons from *Clareza Grade 8: Mathematics*, an eighth grade year a parent runs at the kitchen table. Each lesson is a short script for you, a page to read together, and a practice sheet you may copy for your household. Answers are at the back.

Clareza Grade 8: Mathematics. Sample edition.

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Clareza is a secular curriculum. Standards references: Common Core State Standards and the Next Generation Science Standards.



MATH · UNIT 1

Exponents, Powers, and Scientific Notation

This unit builds exponent notation from scratch: what a power actually means, the three rules for combining powers with the same base, then the strange-but-necessary extension to zero and negative exponents, and finally scientific notation for very big and very small numbers. It ends with comparing unfamiliar real-world quantities by their order of magnitude.

In this unit

1. Evaluating expressions with exponents
2. Simplifying with the exponent rules
3. When exponent rules do not apply
4. Zero and negative exponents from patterns
5. Writing numbers in scientific notation
6. Calculating with scientific notation
7. Comparing quantities by order of magnitude
8. Why zero and negative exponents always work
9. Finding mistakes in exponent solutions

MATH · UNIT 1 · LESSON 1

Evaluating expressions with exponents



Goal: Evaluate numerical expressions with positive integer exponents using order of operations.

About 25 minutes

Teach it FOR THE PARENT

1. **Say:** Quick check: what is 4 x 4 x 4? Now, what do you think 4^3 means? Type your guess.
   Look for: Student connects 4^3 to 4 x 4 x 4 = 64, showing recall of repeated multiplication from prior grades.
2. **Say:** An exponent counts how many times you multiply the base by itself. 2^3 is not 2 x 3. It is 2 x 2 x 2 = 8. Base first, then exponent as the count.
   Look for: Student can state base and exponent roles separately, not confuse exponent with a multiplier.
3. **Say:** What is 5^0? Before you answer, think: does the pattern 5^3, 5^2, 5^1 going down help you guess 5^0?
   Look for: Student says 1, ideally reasoning from the pattern of dividing by 5 each step, not saying 0.
4. **Say:** When exponents mix with other operations, exponents go first. Then multiply or divide. Then add or subtract. Grouping symbols like parentheses still come before everything.
   Look for: Student can restate the order: grouping, then exponents, then multiply/divide, then add/subtract.
5. **Say:** Let's solve 3 + 2^4 together. First: what does 2^4 equal? Then: what do we do with that result and the 3?
   Look for: Student computes 2^4 = 16 first, then adds 3 to get 19, in that order.
6. **Say:** Now: 5 x 3^2 - 4. What's 3^2? Once you have that number, what comes next, multiplying or subtracting?
   Look for: Student finds 3^2 = 9, multiplies 5 x 9 = 45 before subtracting, gets 41.
7. **Say:** Your turn, fresh problem: evaluate 10 - 2^3 + 1. Work it step by step and tell me each step.
   Look for: Student computes 2^3 = 8 first, then works left to right: 10 - 8 = 2, then 2 + 1 = 3.
8. **Say:** You now evaluate exponents inside bigger expressions by doing the power first. Let's practice this on more problems.
   Look for: Student is ready to move into independent practice items.

If they get stuck

- *Multiplying always makes a number bigger and dividing always makes it smaller, so a bigger exponent must give a bigger number.*
  Use a table extending 2^3, 2^2, 2^1, 2^0, 2^-1, 2^-2 side by side and have students describe the pattern in the values, not just the exponents, before naming a rule.
- *In scientific notation, moving the decimal point and changing the exponent are two separate, unrelated steps to memorize.*
  Anchor every conversion to a number line or place-value chart showing the decimal actually sliding across powers-of-ten columns, not a rule about counting digits.
- *Anything to the zero power should equal zero, because zero times anything is zero.*
  Extend the division pattern 2^3/2^3 = 2^0 = 1 explicitly and require students to state the pattern rule in their own words before naming it as a definition.

Read together FOR THE STUDENT

An exponent tells you how many times to multiply a number by itself. In 2^3, the base is 2 and the exponent is 3. That means 2 x 2 x 2 = 8, not 2 x 3. When exponents show up with other operations like + or x, do the exponent first, before multiplying or adding. Think of it like a rule in a game: exponents go before the other moves.

Going further: A power like 5^3 has a base (5) and an exponent (3). The exponent counts repeated multiplication: 5^3 = 5 x 5 x 5 = 125. Order of operations places exponents after grouping symbols, but before multiplication, division, addition, and subtraction. In an expression like 3 x 2^2 + 1, evaluate 2^2 first. Then multiply, then add: 3 x 4 = 12, then 12 + 1 = 13. Any nonzero number to the zero power equals 1, not 0, the exponent counts factors, and zero factors of a number gives you 1 by the pattern of the exponent rules.

Watch it work

Evaluate: 3 + 2^4

1. Find the exponent part first: 2^4 means 2 x 2 x 2 x 2.
2. Multiply step by step: 2 x 2 = 4, 4 x 2 = 8, 8 x 2 = 16.
3. Now the expression is 3 + 16.
4. Add: 3 + 16 = 19.

The idea: Exponents are evaluated before addition, even though addition appears first in the expression.

Evaluate: 5 x 3^2 - 4

1. Find the exponent part first: 3^2 = 3 x 3 = 9.
2. The expression is now 5 x 9 - 4.
3. Multiply before subtracting: 5 x 9 = 45.
4. Subtract: 45 - 4 = 41.

The idea: The exponent applies only to its own base, 3, not to the 5 next to it.

 Name: _____

Practice: Evaluating expressions with exponents

1. What is 2^3?

(A) 6 (B) 8 (C) 9

2. What is 5^0?

(A) 0 (B) 5 (C) 1

3. Evaluate: 4 + 3^2

(A) 49 (B) 10 (C) 13

4. Evaluate: 2^4 - 5

(A) -1 (B) 11 (C) 3

5. Evaluate: 6 x 2^2

(A) 12 (B) 24 (C) 144

6. Evaluate: 3^2 + 4^2

(A) 25 (B) 14 (C) 49

7. Evaluate: 20 - 2^3 + 1

(A) 10 (B) 11 (C) 13

8. You are simplifying an expression and need to find the value of 3^4. What is the value of 3^4?

(A) 64 (B) 81 (C) 12

9. You are simplifying an expression and need to find the value of 9^0. What is the value of 9^0?

(A) 9 (B) 1 (C) 0

10. Evaluate: 10 + 5^2

(A) 225 (B) 20 (C) 35

11. Evaluate: 30 - 2^4 + 4

(A) 26 (B) 10 (C) 18

12. Evaluate: 2 + 3 x 2^3 - 4

(A) 22 (B) 16 (C) 36

13. Evaluate: 8 + 6 x 7^0 - 3^2

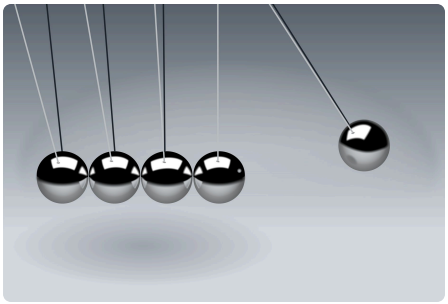
(A) 5 (B) -1 (C) 8

MATH · UNIT 1 · LESSON 2

Simplifying with the exponent rules

Goal: Apply the product-of-powers, quotient-of-powers, and power-of-a-power rules to simplify expressions with a single common base.

About 25 minutes



Teach it FOR THE PARENT

1. **Say:** Quick question: which is bigger, 2^3 or 2^5? ... Right, 2^5, because you're multiplying more 2's together. Today we find a shortcut so you never have to expand a huge string of 2's by hand.
   Look for: Student recognizes more factors of the same base gives a bigger result for bases greater than 1.
2. **Say:** When the base is the SAME, three rules save you work. Multiplying same-base powers: add the exponents. Dividing same-base powers: subtract the exponents. A power raised to a power: multiply the exponents. That's it, three moves, three operations.
   Look for: Student can restate the three rules in their own words without a specific example.
3. **Say:** Watch: 3^4 * 3^5. Same base, 3? Yes. So I add: 4 + 5 = 9. Answer: 3^9. Quick check, did the base change? No, it's still 3. Only the exponent moved.
   Look for: Student confirms the base stayed the same and only exponents were combined.
4. **Say:** Now x^8 / x^3. Same base, x? Yes. Dividing means subtract: 8 - 3 = 5. Answer: x^5. Now (5^2)^3, a power inside a power. Multiply: 2 * 3 = 6. Answer: 5^6.
   Look for: Student applies subtract for division and multiply for power-of-a-power correctly.
5. **Say:** Your turn, fresh problem: simplify 7^6 / 7^2. Same base? What operation do you use? Work it step by step before checking.
   Look for: Student identifies same base 7, subtracts 6 - 2 = 4, writes 7^4.
6. **Say:** Warning: these rules ONLY work when the base is identical. 2^3 * 5^2 cannot be combined this way, different bases, no shortcut. Always check the base first, every single time.
   Look for: Student states that mismatched bases block use of the rules.
7. **Say:** You now have three exponent rules for a single common base. Next up: practice problems. Check the base first, then pick add, subtract, or multiply for the exponents.
   Look for: Student is ready to start independent practice, checking base before choosing an operation.

If they get stuck

- *Multiplying always makes a number bigger and dividing always makes it smaller, so a bigger exponent must give a bigger number.*
  Use a table extending 2^3, 2^2, 2^1, 2^0, 2^-1, 2^-2 side by side and have students describe the pattern in the values, not just the exponents, before naming a rule.
- *In scientific notation, moving the decimal point and changing the exponent are two separate, unrelated steps to memorize.*
  Anchor every conversion to a number line or place-value chart showing the decimal actually sliding across powers-of-ten columns, not a rule about counting digits.
- *Anything to the zero power should equal zero, because zero times anything is zero.*
  Extend the division pattern 2^3/2^3 = 2^0 = 1 explicitly and require students to state the pattern rule in their own words before naming it as a definition.

Read together FOR THE STUDENT

Exponent rules are shortcuts for repeated multiplication with the SAME base. Product rule: x^3 * x^2 = x^(3+2) = x^5, because you're combining 3 x's and 2 x's into 5 x's multiplied together. Quotient rule: x^5 / x^2 = x^(5-2) = x^3, because dividing cancels matching x's. Power-of-a-power: (x^3)^2 = x^(3*2) = x^6, because you're repeating the group 'x^3' two times. Same base only, never combine 2^3 and 5^2 this way.

Going further: When bases match, three rules replace repeated multiplication with addition or subtraction of exponents. Product of powers: a^m * a^n = a^(m+n). Quotient of powers: a^m / a^n = a^(m-n), for a not 0. Power of a power: (a^m)^n = a^(m*n). Each rule comes from expanding the exponents as repeated factors and counting them. These rules only work when the base, the number being raised to a power, is identical in every term.

Watch it work

Simplify: 3^4 * 3^5

1. Check the bases: both are 3, so the product rule applies.
2. Add the exponents: 4 + 5 = 9.
3. Write the result: 3^4 * 3^5 = 3^9.
4. Do not multiply the bases, the base stays 3, it does not become 9.

The idea: Same base, multiplication: add the exponents.

Simplify: x^8 / x^3

1. Check the bases: both are x, so the quotient rule applies.
2. Subtract the exponents: 8 - 3 = 5.
3. Write the result: x^8 / x^3 = x^5.
4. The base x never changes, only the exponent changes.

The idea: Same base, division: subtract the exponents (top minus bottom).

Simplify: (5^2)^3

1. Recognize this as a power raised to another power, so use power-of-a-power.
2. Multiply the exponents: 2 * 3 = 6.
3. Write the result: (5^2)^3 = 5^6.
4. Check by expanding: (5^2)^3 = 5^2 * 5^2 * 5^2 = 5^(2+2+2) = 5^6, which matches.

The idea: A power raised to a power: multiply the exponents.

Practice: Simplifying with the exponent rules

1. Simplify: 2^3 * 2^4

(A) 2^7 (B) 2^12 (C) 4^7

2. Simplify: x^9 / x^4

(A) x^2.25 (B) x^5 (C) x^13

3. Simplify: (4^2)^3

(A) 4^9 (B) 4^6 (C) 4^5

4. Simplify: 5^7 / 5^7

(A) 5^0, which equals 1 (B) 5^14 (C) 5^0, which equals 0

5. Simplify: y^3 * y^3 * y^3

(A) 3y^3 (B) y^9 (C) y^27

6. Simplify: (7^3)^2 / 7^4

(A) 7^10 (B) 7^1 (C) 7^2

7. Which expression is equal to a^5 * a^2 / a^3?

(A) a^10 (B) a^4 (C) a^0

8. A student says 2^5 is bigger than 2^3, and by the same idea, (1/2)^5 must be bigger than (1/2)^2 because 5 > 2. Is this reasoning correct for (1/2)^5 versus (1/2)^2?

(A) Yes, because a bigger exponent always gives a bigger result in every case, no matter what the base happens to be

(B) No, (1/2)^5 is smaller, because multiplying fractions less than 1 repeatedly makes the result smaller

(C) They are equal to each other, since only the two exponents differ and the base of 1/2 stays exactly the same in both of them

9. Simplify: 3^5 * 3^2

(A) 3^7 (B) 9^7 (C) 3^10

10. Simplify: p^10 / p^6

(A) p^16 (B) p^60 (C) p^4

11. Simplify: (k^5)^3

(A) k^8 (B) k^15 (C) k^2

12. Simplify: (x^2)^4 * x^3 / x^5

(A) x^6 (B) x^16 (C) x^4

13. Simplify: (5^3)^2 / (5^2 * 5^4)

(A) 1 (B) 5 (C) 0

14. Dev says multiplying always makes a number bigger, so (0.1)^3 * (0.1)^2 must be bigger than (0.1)^3. Which is correct?

(A) Bigger, because multiplying always makes a number bigger

(B) Smaller, since 0.1^5 is 0.00001 and 0.1^3 is 0.001

(C) Equal, because the base 0.1 is the same in both

When exponent rules do not apply



Goal: Distinguish expressions where an exponent rule applies (same base) from superficially similar expressions where it does not (different bases, or addition instead of multiplication).

About 25 minutes

Teach it FOR THE PARENT

1. **Say:** Quick check: does 2^3 · 2^4 = 2^7, or does 2^3 · 5^4 also collapse into one power like that? Write yes or no for each, and explain why.
   Look for: Student says yes for same base, no for different bases, naming 'same base' as the reason.
2. **Say:** You already know that x^a · x^b = x^(a+b) and x^a ÷ x^b = x^(a-b). Those rules have two conditions hiding inside them: the base must be identical, and the operation must be multiplication or division, never addition or subtraction. Today we hunt for expressions that LOOK like the rule but fail one of those two checks.
   Look for: Nothing to check yet, this sets up the two-question test students will apply.
3. **Say:** Look at 5^3 · 5^6. Step 1: same base? Yes, both 5. Step 2: multiplication? Yes. So the rule fires: 5^(3+6) = 5^9. Now look at 5^3 · 6^6. Step 1: same base? No, 5 and 6. Rule stops right there. We cannot combine it.
   Look for: Student can restate: check base first, then check operation, in that order.
4. **Say:** Now trickier: 8^4 + 8^2. Step 1: same base? Yes, both 8. Step 2: multiplication or division? No, it's addition. Even with matching bases, the rule does not apply. This one stays as 8^4 + 8^2.
   Look for: Student recognizes matching base alone is not sufficient, operation matters too.
5. **Say:** Your turn. Sort these three into 'rule applies' or 'rule does not apply', and say WHY for each: (a) 9^2 · 9^5, (b) 9^2 · 3^5, (c) 9^2 + 9^5.
   Look for: Applies to (a) only, same base 9, multiplication. (b) fails base check. (c) fails operation check.
6. **Say:** You now have a two-question test: same base? multiplying or dividing? Both must be yes. Use that test on every item coming up, don't just look for numbers that seem similar.
   Look for: Ready to move into practice; no new content here.

If they get stuck

- *Multiplying always makes a number bigger and dividing always makes it smaller, so a bigger exponent must give a bigger number.*
  Use a table extending 2^3, 2^2, 2^1, 2^0, 2^-1, 2^-2 side by side and have students describe the pattern in the values, not just the exponents, before naming a rule.
- *In scientific notation, moving the decimal point and changing the exponent are two separate, unrelated steps to memorize.*
  Anchor every conversion to a number line or place-value chart showing the decimal actually sliding across powers-of-ten columns, not a rule about counting digits.
- *Anything to the zero power should equal zero, because zero times anything is zero.*
  Extend the division pattern 2^3/2^3 = 2^0 = 1 explicitly and require students to state the pattern rule in their own words before naming it as a definition.

Read together FOR THE STUDENT

Exponent rules like x^a times x^b = x^(a+b) only work when the base, the number being multiplied, is the SAME on both sides. 3^2 times 3^4 works: same base, 3. But 3^2 times 5^4 does not: different bases. Also, the rule is for MULTIPLYING powers, not adding. 3^2 + 3^4 is not 3^6, it's just two numbers added. Always check: same base? Multiplying, not adding? Only then does the shortcut apply.

Going further: The product rule x^a · x^b = x^(a+b) and quotient rule x^a ÷ x^b = x^(a-b) require identical bases, because they come from counting repeated factors of that one base. If bases differ (2^3 · 5^2), there is no shared factor to combine, you must compute each power separately. If the operation is addition or subtraction instead of multiplication or division (x^a + x^b), there is no rule at all; exponents don't combine over addition. Check base, then check operation, before applying any rule.

Watch it work

Simplify if possible: 7^3 · 7^5

1. Check the base: both terms use base 7, same base.
2. Check the operation: multiplication, rule applies.
3. Add the exponents: 7^3 · 7^5 = 7^(3+5) = 7^8.
4. Result: 7^8 (do not multiply out, keep as a power).

The idea: Same base AND multiplication together unlock the product rule.

Simplify if possible: 4^6 + 4^2

1. Check the base: both terms use base 4, same base, so far so good.
2. Check the operation: this is addition, not multiplication.
3. The product rule only combines exponents under multiplication, it does not apply here.
4. Result: 4^6 + 4^2 cannot be simplified into a single power. Compute each term separately if a number is needed.

The idea: Same base is not enough, the operation must also be multiplication or division.

 Name: _____

Practice: When exponent rules do not apply

1. Which expression can be simplified using the product rule (same base, exponents add)?

(A) 6^3 + 6^5 (B) 6^3 · 6^5 (C) 6^3 · 7^5

2. Which expression can be simplified using the quotient rule (same base, exponents subtract)?

(A) 10^7 ÷ 4^2 (B) 10^7 ÷ 10^2 (C) 10^7 − 10^2

3. A student says (1/2)^5 must be bigger than (1/2)^2 because 5 is a bigger exponent than 2. What's wrong with this reasoning?

(A) Multiplying a fraction between 0 and 1 by itself repeatedly makes it smaller, not bigger, so more multiplications means a smaller result.

(B) Nothing, bigger exponents always give bigger results.

(C) The exponent rule doesn't apply here at all because there's only one base in each expression, so neither one can be compared to the other one.

4. Which of these CANNOT be simplified with a single exponent rule?

(A) 3^9 · 3^4 (B) 3^9 ÷ 3^4 (C) 3^9 + 3^4

5. Look at this pattern of powers of 5: 5^2 = 25, then 5^1 = 5, and next comes 5^0. A student writes 5^0 = 0. What is 5^0 actually, and why?

(A) 1, because each step down divides by 5, and 5 ÷ 5 = 1.

(B) 5, because the exponent doesn't change the base when it's 0.

(C) 0, because zero times anything is zero.

6. Sort this pair correctly: x^6 · x^3 and x^6 · y^3. Which statement is true?

(A) Neither of the two expressions simplifies, because both of them have variables as bases instead of numbers.

(B) Only x^6 · x^3 simplifies, into x^9; x^6 · y^3 stays as is because the bases differ.

(C) Both of the expressions simplify in exactly the same way, because the exponents match up in each one.

7. Which product can be written as a single power of one base using the product rule?

(A) 5^2 · 3^3 (B) 7^2 · 6^7 (C) 4^3 · 4^6

8. Which expression can be rewritten as 9^7 using an exponent rule?

(A) 9^3 · 7^4 (B) 9^3 · 9^4 (C) 9^3 + 9^4

9. A student writes 2^3 + 2^4 = 2^7 because "you add the exponents." What is wrong with this reasoning?

(A) The rule is for multiplying. Here 2^3 + 2^4 = 8 + 16 = 24.

(B) Adding makes the result bigger, so it must exceed 2^7.

(C) The exponents should be multiplied, giving 2^12.

10. A student simplifies 8^3 · 8^0 and gets 0, since "anything to the zero power is zero." What is the correct simplified result?

(A) 1, because the zero exponent makes the whole product 1.

(B) 8^3, because 8^0 = 1 and 8^(3+0) = 8^3.

(C) 0, because 8^0 = 0 and 0 times anything is 0.

11. Which expression can be written as a single power of 2?

(A) 2^5 · 5^2 (B) 4 · 2^5 (C) 2^5 + 2^3

12. Which is the best simplification of x^3 · x^4 + y^3 · y^4, where x and y are different positive numbers?

(A) (xy)^7 (B) (x + y)^7 (C) x^7 + y^7

MATH · UNIT 1 · LESSON 4

Zero and negative exponents from patterns



Goal: Explain why extending the pattern of decreasing exponents forces the definitions a^0 = 1 and a^-n = 1/a^n.

About 25 minutes

Teach it FOR THE PARENT

1. **Say:** Quick question: is 2^-3 bigger or smaller than 2^3? Don't calculate yet. Just guess, and tell me why you think that.
   Look for: Student states a guess and a reason, likely revealing the 'bigger exponent = bigger number' misconception.
2. **Say:** Let's build a table of powers of 4: 4^3=64, 4^2=16, 4^1=4. Look at what happens each step down. What operation connects 64 to 16, and 16 to 4?
   Look for: Student identifies 'divide by 4' as the step-down operation.
3. **Say:** If that pattern keeps going, what should 4^0 be? Use the divide-by-4 rule to find it.
   Look for: Student divides 4÷4 and gets 4^0=1, not 0.
4. **Say:** Now let's use the quotient rule instead of a table. We know a^m ÷ a^n = a^(m-n). Watch: 6^2 ÷ 6^2 = 6^0, and 6^2 ÷ 6^2 is also just 1. So 6^0=1. Same idea, no table needed.
   Look for: Student follows both routes (table and quotient rule) landing on the same answer.
5. **Say:** Now negative exponents: 6^0 ÷ 6^1 = 6^-1. We just found 6^0=1, so 6^-1 = 1÷6 = 1/6. Same pattern, one more step down.
   Look for: Student sees a^-1=1/a comes from continuing the same division pattern.
6. **Say:** Your turn, new base: use the quotient rule to explain why 9^0=1 and 9^-2=1/81. Show both steps like I did.
   Look for: Student sets m=n for the zero case and m=0,n=2 for the negative case, gets 1 and 1/81.
7. **Say:** The base never mattered, only the pattern did. That's why this works for ANY nonzero base. Let's practice explaining this with different numbers.
   Look for: Student is ready to generalize the argument to bases not yet tabulated.

If they get stuck

- *Multiplying always makes a number bigger and dividing always makes it smaller, so a bigger exponent must give a bigger number.*
  Use a table extending 2^3, 2^2, 2^1, 2^0, 2^-1, 2^-2 side by side and have students describe the pattern in the values, not just the exponents, before naming a rule.
- *In scientific notation, moving the decimal point and changing the exponent are two separate, unrelated steps to memorize.*
  Anchor every conversion to a number line or place-value chart showing the decimal actually sliding across powers-of-ten columns, not a rule about counting digits.
- *Anything to the zero power should equal zero, because zero times anything is zero.*
  Extend the division pattern 2^3/2^3 = 2^0 = 1 explicitly and require students to state the pattern rule in their own words before naming it as a definition.

Read together FOR THE STUDENT

Look at powers of 3: 3^3=27, 3^2=9, 3^1=3. Each step down, you divide by 3. If the pattern keeps going, 3^0 must be 3÷3=1. Keep dividing: 3^-1 = 1÷3 = 1/3. That is why a^0=1 and negative exponents give fractions. It is not a new rule. It is the same pattern, continued.

Going further: A list of powers of a base shows each step down divides by the base. That is the quotient rule you already know: a^m ÷ a^n = a^(m-n). Keep the pattern past exponent 1: a^1 ÷ a = a^0, so a^0 = a/a = 1. Divide once more: a^0 ÷ a = a^-1, so a^-1 = 1/a. This works for any base a (a≠0), because the division pattern never depends on which base you picked.

Watch it work

Extend the pattern of powers of 5 below exponent 1: 5^3=125, 5^2=25, 5^1=5, 5^0=?, 5^-1=?

1. Look at the list of powers above. Each step down divides by 5: 125÷5=25, 25÷5=5.
2. Keep dividing by 5 to continue the same pattern: 5÷5=1. So 5^0=1.
3. Divide by 5 again to go one more step down: 1÷5=1/5. So 5^-1=1/5.
4. Check with the quotient rule: 5^1÷5^1=5^(1-1)=5^0, and 5^1÷5^1=1. Both methods agree.

The idea: Zero and negative exponents are just the pattern of dividing by the base, continued past exponent 1.

Use the quotient rule to explain why 7^0=1 and 7^-2=1/49, without a table.

1. The quotient rule says a^m ÷ a^n = a^(m-n) for any exponents m and n.
2. Let m=n=3: 7^3 ÷ 7^3 = 7^(3-3) = 7^0. Since any number divided by itself is 1, 7^0=1.
3. Let m=0, n=2: 7^0 ÷ 7^2 = 7^(0-2) = 7^-2.
4. Substitute 7^0=1: 1 ÷ 7^2 = 7^-2, so 7^-2 = 1/49.

The idea: The quotient rule forces a^0=1 and a^-n=1/a^n for any nonzero base, not just the one in a table.

Practice: Zero and negative exponents from patterns

1. Powers of 2: 2^4=16, 2^3=8, 2^2=4. Each step down divides by 2. What is 2^1?

(A) 0 (B) 2 (C) 8

2. Using the same divide-by-2 pattern, what is 2^0?

(A) 0 (B) 1 (C) 2

3. Continuing the pattern below 2^0=1, what is 2^-1?

(A) 2 (B) 1/2 (C) -2

4. Which comparison is true?

(A) 3^-2 is smaller than 3^2 (B) 3^-2 is bigger than 3^2, since -2 is farther from zero

(C) 3^-2 equals 3^2 because exponents just flip sign

5. Use the quotient rule a^m ÷ a^n = a^(m-n). What does 5^4 ÷ 5^4 equal, written as a power of 5?

(A) 5^8 (B) 5^0 (C) 5^1

6. 5^4 ÷ 5^4 also equals 1, since any nonzero number divided by itself is 1. What does this tell you about 5^0?

(A) 5^0 must equal 0, since the exponent is 0

(B) 5^0 could be any number, since it's undefined (C) 5^0 must equal 1

7. Using 7^0=1 and the quotient rule 7^0÷7^3=7^-3, what is 7^-3?

(A) 343 (B) -343 (C) 1/343

8. A student says: 'a^0=1 only works for bases like 2, 5, and 7 that have a list of powers written out.' What's wrong with this claim?

(A) It only works for bases that are bigger than 1, so the student is right that fractions and small bases are left out

(B) Nothing is wrong with the claim; each base needs its own table of powers to confirm that a^0=1 holds for it

(C) The quotient rule argument uses no specific base, so it works for any nonzero base

9. Powers of 10: 10^4 = 10000, 10^3 = 1000, 10^2 = 100. Each step down divides by 10. What is 10^1?

(A) 1 (B) 1000 (C) 10

10. Powers of 8: 8^3 = 512, 8^2 = 64, 8^1 = 8. Each step down divides by 8. What is 8^0?

(A) 0 (B) 1 (C) 8

11. Powers of 10 keep stepping down: 10^1 = 10, 10^0 = 1, 10^-1 = 1/10. Continue the same pattern. What is 10^-2?

(A) 1/20 (B) -100 (C) 1/100

12. A student says 6^0 = 0 because zero times anything is zero. Which reasoning correctly settles what 6^0 is?

(A) 6^2 minus 6^2 is 0, so the zero power must also be 0

(B) 6^0 means 6 multiplied by 0, and zero times anything is 0

(C) 6^2 ÷ 6^2 = 6^0, and 36 ÷ 36 = 1, so 6^0 = 1

13. Using 2^-5 = 1/32 from the divide-by-2 pattern, what is 2^3 × 2^-5, written as a single fraction or number?

(A) 4 (B) 1/4 (C) 1/32

14. A student writes 0.0072 = 7.2 × 10^-3 and says the -3 is just a rule to memorize. Which explanation shows it follows the exponent pattern?

(A) 10^0 equals 0, so the exponent counts down from there to -3 and makes the number tiny

(B) 7.2 ÷ 10 three times gives 0.72, 0.072, 0.0072, and each divide-by-10 step lowers the exponent by 1

(C) Moving the point three places and choosing the exponent are two separate steps with no link between them

Writing numbers in scientific notation

Goal: Convert numbers between standard form and scientific notation for very large and very small numbers.

About 25 minutes



Teach it FOR THE PARENT

1. **Say:** The distance to the sun is about 93,000,000 miles. A red blood cell is about 0.000008 meters wide. Both numbers are a pain to write and read. Scientists needed a shorthand. Today you learn it.
   Look for: Attention on the problem of unwieldy numbers, no answer expected yet.
2. **Say:** Scientific notation is a x 10^n. The part 'a' is a decimal with exactly one nonzero digit before the point, so 1 to 9 point something. The exponent n counts how many places the decimal point moved to get there. Left move on a big number gives positive n. Right move on a small number gives negative n.
   Look for: Student can state that a has one leading nonzero digit and n counts decimal moves.
3. **Say:** Watch: 4,500,000. Where's the decimal point? After the last zero. Move it left until one digit is left in front: 4.5. Count the moves: 6. Big number, moved left, so exponent is positive 6. Answer: 4.5 x 10^6. Why 6 and not some other number?
   Look for: Student can explain that 6 is the count of decimal-point moves, not a memorized fact.
4. **Say:** Your turn to check me: does 5^0 equal zero? Look at this pattern: 5^2=25, 5^1=5, 5^0=?. Each step right, we divide by 5. So 5^0 = 5 divided by 5. What is that?
   Look for: Student answers 1, reasoning from the division pattern, not from 'zero times anything'.
5. **Say:** Now a small number: 0.00032. Move the decimal right until one digit is in front: 3.2. Count the moves: 4. Small number, moved right, so exponent is negative 4. Answer: 3.2 x 10^-4.
   Look for: Student notices direction of the move matches the sign of the exponent.
6. **Say:** Quick question: is 2^-3 bigger or smaller than 2^3? Don't just compare the exponents 3 and -3. Think about what a negative exponent means to the size of the number.
   Look for: Student says 2^-3 is smaller, connecting negative exponent to a small fraction, not to 'bigger exponent wins'.
7. **Say:** Now reverse it. Convert 7.1 x 10^5 back to standard form. The exponent is positive 5, so move the decimal 5 places right. Add zeros as needed. What do you get?
   Look for: Student moves decimal right 5 places, getting 710,000.
8. **Say:** Your turn, fresh number. Convert 62,000 to scientific notation. Then convert 8.3 x 10^-2 back to standard form. Show the decimal move and the count for each.
   Look for: Correct answers 6.2 x 10^4 and 0.083, with decimal move and count stated for both.

9. **Say:** You've got both moves now. Big numbers get a positive exponent. Small numbers get a negative exponent. Time to practice on your own.
   Look for: Ready to move into independent practice items.

If they get stuck

- *Multiplying always makes a number bigger and dividing always makes it smaller, so a bigger exponent must give a bigger number.*
  Use a table extending 2^3, 2^2, 2^1, 2^0, 2^-1, 2^-2 side by side and have students describe the pattern in the values, not just the exponents, before naming a rule.
- *In scientific notation, moving the decimal point and changing the exponent are two separate, unrelated steps to memorize.*
  Anchor every conversion to a number line or place-value chart showing the decimal actually sliding across powers-of-ten columns, not a rule about counting digits.
- *Anything to the zero power should equal zero, because zero times anything is zero.*
  Extend the division pattern 2^3/2^3 = 2^0 = 1 explicitly and require students to state the pattern rule in their own words before naming it as a definition.

Read together FOR THE STUDENT

Scientific notation writes a number as a decimal times a power of 10. The decimal is between 1 and 10. Example: 4,500,000 = 4.5 x 10^6. Think of the exponent as a counter. It counts how many places the decimal point moved. Big numbers get a positive exponent. Tiny numbers, like 0.0003, get a negative exponent. It's like a shorthand for numbers with lots of zeros.

Going further: Scientific notation is a x 10^n, where a has one nonzero digit before the decimal point, so 1 <= a < 10. To convert standard form, move the decimal point until only one nonzero digit remains in front. Count the moves. That count is n. Moving left (big number) gives positive n. Moving right (small number) gives negative n. To reverse it, move the decimal point n places, using the exponent's sign to pick the direction.

Watch it work

Convert 4,500,000 to scientific notation.

1. Find the decimal point. It's after the last 0: 4500000.
2. Move it left until one nonzero digit remains before it: 4.5.
3. Count the moves: 6 places left.
4. Moving left on a big number means a positive exponent: 4.5 x 10^6.

The idea: The exponent counts how many places the decimal point moved to reach one leading digit.

Convert 0.00032 to scientific notation.

1. Find the decimal point: 0.00032.
2. Move it right until one nonzero digit remains before it: 3.2.
3. Count the moves: 4 places right.
4. Moving right on a small number means a negative exponent: 3.2 x 10^-4.

The idea: Small numbers under 1 always get a negative exponent when written in scientific notation.

Convert 7.1 x 10^5 back to standard form.

1. Read the exponent: 5. It is positive.
2. A positive exponent means move the decimal point right.
3. Move it 5 places right, adding zeros as placeholders: 710000.
4. Check: 7.1 x 10^5 = 710,000.

The idea: A positive exponent always undoes into a decimal move to the right, making the number bigger.

Clareza Grade 8 · Math Lesson 5 Name: _____

Practice: Writing numbers in scientific notation

1. Convert 6,000 to scientific notation.

(A) 6 x 10^-3 (B) 6 x 10^0 (C) 6 x 10^3 (D) 0.6 x 10^4

2. Convert 0.007 to scientific notation.

(A) 7 x 10^-2 (B) 7 x 10^-3 (C) 0.7 x 10^-2 (D) 7 x 10^3

3. Convert 3.4 x 10^5 back to standard form.

(A) 340,000 (B) 3,400,000 (C) 34,000 (D) 0.000034

4. Convert 9.2 x 10^-4 back to standard form.

(A) 0.000092 (B) 0.00092 (C) 92,000 (D) 0.0092

5. Convert 810,000,000 to scientific notation.

(A) 8.1 x 10^8 (B) 81 x 10^7 (C) 8.1 x 10^-8 (D) 8.1 x 10^7

6. Which is bigger: 4 x 10^-5 or 4 x 10^3?

(A) They are equal, because both start with 4

(B) 4 x 10^-5, because the digit 5 in the exponent is bigger than the digit 3

(C) 4 x 10^3, because a negative exponent makes a small number

7. What does 6^0 equal?

(A) 6 (B) 0 (C) 1

8. Convert 5.03 x 10^6 back to standard form.

(A) 0.00000503 (B) 5,300,000 (C) 503,000 (D) 5,030,000

9. Convert 70,000 to scientific notation.

(A) 7 x 10^-4 (B) 7 x 10^5 (C) 7 x 10^4 (D) 70 x 10^3

10. A grain of pollen is 0.0002 meters wide. Write 0.0002 in scientific notation.

(A) 2 x 10^4 (B) 0.2 x 10^-3 (C) 2 x 10^-4 (D) 2 x 10^-3

11. Convert 0.0000375 to scientific notation.

(A) 3.75 x 10^-4 (B) 37.5 x 10^-6 (C) 3.75 x 10^-5 (D) 3.75 x 10^5

12. What is the value of 2.5 x 10^0?

(A) 2.5 (B) 25 (C) 0.25 (D) 0

13. Which of these numbers is the greatest?

(A) 7 x 10^-3 (B) 2 x 10^-1 (C) 4 x 10^-6 (D) 9 x 10^-2

14. Rewrite 0.62 x 10^-3 in proper scientific notation.

(A) 6.2 x 10^-3 (B) 62 x 10^-5 (C) 6.2 x 10^-4 (D) 6.2 x 10^-2

MATH · UNIT 1 · LESSON 6

Calculating with scientific notation



Goal: Perform addition, subtraction, multiplication, and division of numbers expressed in scientific notation, including cases requiring re-normalization of the leading digit.

About 28 minutes

Teach it FOR THE PARENT

1. **Say:** Quick retrieval: what is 3.4 x 10^2 written as a regular number? What is 7,200,000 in scientific notation? Answer both before you continue.
   Look for: 340 and 7.2 x 10^6, recalled without hesitation.
2. **Say:** Today we add, subtract, multiply, and divide numbers already in scientific notation. Adding and subtracting need matching exponents first. Multiplying and dividing use the exponent laws you already learned: add exponents when multiplying, subtract when dividing.
   Look for: Student can state which operations need matching exponents and which do not.
3. **Say:** Here is a worked example: (3.2 x 10^5) + (4.5 x 10^6). First I make the exponents match. I rewrite 3.2 x 10^5 as 0.32 x 10^6. Now I add the coefficients: 0.32 + 4.5. What do I get?
   Look for: 4.82, leading to 4.82 x 10^6.
4. **Say:** Why couldn't I just add 3.2 and 4.5 right away, before matching exponents?
   Look for: Student explains the exponents represent different place values, so coefficients aren't comparable yet.
5. **Say:** Now multiplication: (6 x 10^4) x (5 x 10^3). Multiply coefficients: 6 x 5 = 30. Add exponents: 4 + 3 = 7. So we get 30 x 10^7. Is 30 a valid coefficient?
   Look for: No, coefficient must be between 1 and 10, so it needs renormalizing to 3.0 x 10^8.
6. **Say:** When I shifted 30 to 3.0, I moved the decimal one place left. Why did the exponent go UP by 1, not down?
   Look for: Shifting the decimal left makes the coefficient smaller, so the exponent must grow to keep the value the same.
7. **Say:** Your turn, fresh problem: (2.4 x 10^6) divided by (8 x 10^2). Divide the coefficients, subtract the exponents, and check if you need to renormalize.
   Look for: 2.4 / 8 = 0.3, exponent 6 - 2 = 4, raw result 0.3 x 10^4, renormalized to 3 x 10^3.
8. **Say:** You've done all four operations now. The practice set mixes them. Read the operation sign first, that decides your whole method. Then check your final coefficient every time.
   Look for: Student moves into practice checking the operation before starting each problem.

If they get stuck

- *Multiplying always makes a number bigger and dividing always makes it smaller, so a bigger exponent must give a bigger number.*
  Use a table extending 2^3, 2^2, 2^1, 2^0, 2^-1, 2^-2 side by side and have students describe the pattern in the values, not just the exponents, before naming a rule.
- *In scientific notation, moving the decimal point and changing the exponent are two separate, unrelated steps to memorize.*
  Anchor every conversion to a number line or place-value chart showing the decimal actually sliding across powers-of-ten columns, not a rule about counting digits.
- *Anything to the zero power should equal zero, because zero times anything is zero.*
  Extend the division pattern 2^3/2^3 = 2^0 = 1 explicitly and require students to state the pattern rule in their own words before naming it as a definition.

Read together FOR THE STUDENT

Scientific notation writes numbers as a decimal between 1 and 10, times a power of 10. To add or subtract, the powers of 10 must match first, like adding feet and inches, you convert to the same unit. To multiply, multiply the front numbers and add the exponents. To divide, divide the front numbers and subtract the exponents. Sometimes your answer's front number is too big or too small. Then you shift the decimal and adjust the exponent to fix it, like regrouping in addition.

Going further: Scientific notation is a x 10^n, where 1 <= a < 10. For addition and subtraction, rewrite both numbers with the same exponent, then add or subtract the coefficients. For multiplication, multiply coefficients and add exponents using the product rule. For division, divide coefficients and subtract exponents using the quotient rule. If the coefficient lands outside [1, 10), renormalize. Shift the decimal one place and adjust the exponent by 1 in the opposite direction. Always renormalize last, after the arithmetic is done.

Watch it work

Add: (3.2 x 10^5) + (4.5 x 10^6)

1. Exponents differ (5 and 6), so rewrite one term to match the other.
2. 3.2 x 10^5 = 0.32 x 10^6
3. Add coefficients now that exponents match: 0.32 + 4.5 = 4.82
4. Result: 4.82 x 10^6. Coefficient is already in [1, 10), so no renormalizing needed.

The idea: You cannot add coefficients until the exponents are the same.

Multiply: (6 x 10^4) x (5 x 10^3), then renormalize.

1. Multiply coefficients: 6 x 5 = 30
2. Add exponents: 4 + 3 = 7
3. Raw result: 30 x 10^7
4. 30 is not in [1, 10). Shift the decimal one place left: 30 -> 3.0
5. Each left shift adds 1 to the exponent: 7 + 1 = 8
6. Final answer: 3.0 x 10^8

The idea: Do the arithmetic first, then fix the coefficient back into [1, 10) as a separate last step.

Practice: Calculating with scientific notation

1. Multiply: (2 x 10^3) x (3 x 10^2). What is the result?

(A) 5 x 10^5 (B) 6 x 10^6 (C) 6 x 10^5

2. Divide: (8 x 10^7) / (2 x 10^3). What is the result?

(A) 6 x 10^4 (B) 4 x 10^4 (C) 4 x 10^10

3. Add: (2.1 x 10^4) + (3.4 x 10^4). What is the sum?

(A) 5.5 x 10^4 (B) 5.5 x 10^8 (C) 0.7 x 10^0

4. Subtract: (7.8 x 10^5) - (2.3 x 10^5). What is the result?

(A) 5.5 x 10^0 (B) 5.5 x 10^5 (C) 10.1 x 10^5

5. Add: (5.6 x 10^3) + (2.1 x 10^4). What is the sum?

(A) 2.66 x 10^4 (B) 7.7 x 10^7 (C) 7.7 x 10^4

6. Multiply: (4 x 10^6) x (5 x 10^2). What is the result, fully normalized?

(A) 2 x 10^8 (B) 20 x 10^8 (C) 2 x 10^9

7. Divide: (1.5 x 10^4) / (5 x 10^6). What is the result, fully normalized?

(A) 3 x 10^-3 (B) 0.3 x 10^-2 (C) 3 x 10^-2

8. A student says 4^0 must equal 0, since 0 times anything is 0. What is actually true, and why?

(A) 4^0 = 1, because dividing 4^n by 4^n always gives 1

(B) 4^0 = 4, because the exponent does not change the base

(C) 4^0 = 0, because the exponent means there is nothing there

9. Multiply: (3 x 10^4) x (2 x 10^3). What is the result?

(A) 5 x 10^7 (B) 6 x 10^7 (C) 6 x 10^12

10. Divide: (9 x 10^8) / (3 x 10^5). What is the result?

(A) 3 x 10^3 (B) 6 x 10^3 (C) 3 x 10^13

11. Subtract: (9.6 x 10^3) - (4.1 x 10^3). What is the result?

(A) 5.5 x 10^6 (B) 5.5 x 10^3 (C) 5.5 x 10^0

12. Add: (3.2 x 10^5) + (4.5 x 10^4). What is the sum, fully normalized?

(A) 36.5 x 10^4 (B) 7.7 x 10^5 (C) 3.65 x 10^5

13. Divide: (2.4 x 10^-2) / (8 x 10^-6). What is the result, fully normalized?

(A) 3 x 10^4 (B) 0.3 x 10^4 (C) 3 x 10^3

14. A student multiplies (3 x 10^5) x (2 x 10^-5). What is the product, and why?

(A) 0, because 5 + (-5) = 0 and 10^0 = 0 (B) 6 x 10^10, because 5 - (-5) = 10

(C) 6, because 5 + (-5) = 0 and 10^0 = 1

Comparing quantities by order of magnitude



Goal: Compare the order of magnitude of two unfamiliar real-world quantities given in different units, without being told which quantities to compare or which operation applies.

About 30 minutes

Teach it FOR THE PARENT

1. **Say:** A bacterium is about 0.000002 meters long. The Eiffel Tower is about 300 meters tall. How many times bigger is the tower? Don't calculate yet, just guess: is it hundreds of times, thousands, or millions of times bigger?
   Look for: A guess, even a rough one, showing they sense huge scale difference without computing exactly.
2. **Say:** To compare sizes like this, we write both numbers in scientific notation, a x 10^n. Then we compare the exponents n. A bigger exponent means a bigger number, that's the whole trick. The messy front number (a) almost never changes who wins.
   Look for: Recognition that exponent comparison is the core method, coefficient is secondary.
3. **Say:** Quick check: which is bigger, 3 x 10^5 or 8 x 10^2? Tell me the exponents you're comparing, not just the answer.
   Look for: States 5 vs 2, picks 3 x 10^5, ignores that 8 > 3.
4. **Say:** Let's compare a grain of sand (6 x 10^-4 g) and a raindrop (2 x 10^-2 g). Step 1: same units already. Step 2: compare exponents -4 and -2. Which is bigger? Step 3: the raindrop wins, by about 10^2.
   Look for: Correctly identifies -2 > -4 and states raindrop is about 100 times heavier.
5. **Say:** Watch out: sometimes the units don't match yet, like grams versus kilograms, or minutes versus seconds. You have to convert to the SAME unit before you can compare exponents. Skipping this step gives a wrong answer even if your exponent rule is right.
   Look for: Understands unit conversion must happen before exponent comparison.
6. **Say:** New problem, no hints: A tardigrade is about 3 x 10^-4 m long. A football field is about 1 x 10^2 m long. Which is bigger, and by roughly how many orders of magnitude? Show your exponent comparison.
   Look for: Compares -4 vs 2, states field is about 10^6 times longer, shows exponent subtraction or reasoning.
7. **Say:** You just compared two totally unfamiliar things using only their exponents. That's the whole skill: convert units if needed, write scientific notation, compare exponents. Now try it on your own with new quantities, no examples will match these exactly.
   Look for: Readiness to apply the method to new, unseen comparisons in the practice set.

If they get stuck

- *Multiplying always makes a number bigger and dividing always makes it smaller, so a bigger exponent must give a bigger number.*
  Use a table extending 2^3, 2^2, 2^1, 2^0, 2^-1, 2^-2 side by side and have students describe the pattern in the values, not just the exponents, before naming a rule.
- *In scientific notation, moving the decimal point and changing the exponent are two separate, unrelated steps to memorize.*
  Anchor every conversion to a number line or place-value chart showing the decimal actually sliding across powers-of-ten columns, not a rule about counting digits.
- *Anything to the zero power should equal zero, because zero times anything is zero.*
  Extend the division pattern 2^3/2^3 = 2^0 = 1 explicitly and require students to state the pattern rule in their own words before naming it as a definition.

Read together FOR THE STUDENT

Order of magnitude means 'about how big' a number is, measured in powers of 10. A virus is about 10^-7 meters. A city is about 10^4 meters wide. To compare them, write both in scientific notation, then just compare the exponents. The number with the bigger exponent is bigger, no matter how messy the front number looks. You don't need to calculate the exact size, just the power of 10.

Going further: To compare two quantities' order of magnitude, convert both to scientific notation (a x 10^n, 1≤a<10), matching units first if needed. Then compare exponents n. A larger exponent means a larger quantity, regardless of the coefficient a. If exponents are equal, compare coefficients. This works because each increase of 1 in the exponent means the quantity is about 10 times bigger.

Watch it work

A grain of sand has a mass of about 6 x 10^-4 g. A raindrop has a mass of about 2 x 10^-2 g. Which is heavier, and by roughly what order of magnitude?

1. Both are already in scientific notation with the same unit (grams), so no conversion is needed.
2. Compare exponents: -2 is greater than -4, since -2 is farther right on the number line.
3. So the raindrop (10^-2) is about two orders of magnitude bigger than the grain of sand (10^-4).
4. Answer: the raindrop is heavier, by roughly 10^2 (about 100 times).

The idea: When units already match, compare exponents directly, the coefficient rarely changes which is bigger by orders of magnitude.

A hummingbird's heart beats about 1,200 times per minute. A whale's heart beats about 6 times per minute. Compare these using scientific notation and orders of magnitude.

1. Write both in scientific notation: 1,200 = 1.2 x 10^3, and 6 = 6 x 10^0.
2. Units already match (beats per minute), so compare exponents directly: 3 vs 0.
3. The difference in exponents is 3, meaning the hummingbird's rate is about 10^3 (1,000) times larger.
4. Answer: the hummingbird's heart rate is about three orders of magnitude greater than the whale's.

The idea: Even everyday numbers without units of measurement can be compared by order of magnitude once written in scientific notation.

 Name: _____

Practice: Comparing quantities by order of magnitude

1. Which number is greater: 4 x 10^6 or 9 x 10^3?

(A) 4 x 10^6, because 6 is a bigger exponent than 3

(B) 9 x 10^3, because 9 is a bigger front number

(C) They are equal since both start with single digits

2. A cell has a diameter of about 2 x 10^-5 m. A pinhead has a diameter of about 1 x 10^-3 m. Which is bigger?

(A) The pinhead, because -3 is greater than -5

(B) Cannot tell without converting units first

(C) The cell, because -5 is farther from zero than -3

3. Which is larger: 5^0 or 5^-2?

(A) 5^0, because it equals 1 and 5^-2 equals a fraction less than 1

(B) 5^0, because 0 means nothing so it must be the smallest

(C) 5^-2, because negative exponents always make numbers bigger

4. A dust mite is about 3 x 10^-4 m long. A grain of rice is about 7 x 10^-3 m long. About how many times longer is the grain of rice?

(A) About 2 times longer, since 7 is a bit more than 3

(B) About 10 times longer

(C) About 100 times longer

5. A running ant covers about 4 x 10^1 cm per second. A cheetah covers about 3 x 10^3 cm per second. Which is faster and by roughly what factor?

(A) The ant, because 4 is a bigger digit than 3 in front

(B) The cheetah, by about 10^2 (roughly 100 times)

(C) The cheetah, by about 10^4 times

6. A bacterium's mass is about 1 x 10^-15 kg. A single grain of table salt has a mass of about 5 x 10^-5 g. Convert the salt grain to kilograms, then compare. Which is heavier?

(A) The salt grain, since 5 x 10^-5 g equals 5 x 10^-8 kg, and -8 is greater than -15

(B) The bacterium, since -15 is a smaller exponent, and smaller exponents mean heavier objects

(C) The two cannot be compared at all because the units started out different and cannot be changed

7. Which is greater: 10^0 or 10^-4?

(A) 10^-4, because anything to the zero power is zero

(B) 10^-4, because 4 is a bigger number than 0

(C) 10^0, because it is 1 and 10^-4 is 0.0001

8. A human hair is about 8 x 10^-5 m wide. A red blood cell is about 7 x 10^-6 m wide. Which is wider?

(A) The hair, because -5 is a bigger exponent than -6

(B) Neither, because negative exponents make lengths negative

(C) The red blood cell, because 6 is bigger than 5

9. A grain of pollen is 0.00005 m across. A grain of sand is 4 x 10^-4 m across. Which is bigger?

(A) The pollen, because 0.00005 has more digits than 4

(B) The pollen, because 0.00005 is 5 x 10^-4 and 5 > 4

(C) The sand, because 0.00005 is 5 x 10^-5 and -5 < -4

10. A mouse has a mass of about 2 x 10^1 g. An elephant has a mass of about 5 x 10^6 g. About how many times as massive is the elephant?

(A) About 2.5 x 10^7 times (B) About 2.5 x 10^5 times (C) About 2.5 x 10^6 times

11. A human hair is about 7 x 10^-5 m wide. A virus is about 1 x 10^2 nm wide, where 1 nm = 10^-9 m. Which is wider, and by about how many times?

(A) The hair, by about 7 x 10^-2 times (B) The virus, because 10^2 beats 10^-5

(C) The hair, by about 7 x 10^2 times

12. A bag of flour has a mass of about 2 x 10^0 kg. A grain of sand has a mass of about 5 x 10^-4 g. Use 1 kg = 10^3 g. Which is heavier, and by about how many times?

(A) The flour, by about 4 x 10^6 times (B) The flour, by about 4 x 10^3 times

(C) The sand grain, because 10^0 = 0 makes the flour 0 kg

Why zero and negative exponents always work



Goal: Generalize the exponent rules to justify why a rule that holds for positive integer exponents must also hold for exponent zero and negative integers, using an argument that would apply to any base.

About 28 minutes

Teach it FOR THE PARENT

1. **Say:** Earlier you saw that 2^0=1 and 2^-1=1/2 by dividing by 2 each time the exponent dropped: 2^3=8, 2^2=4, 2^1=2, then 2^0=1, then 2^-1=1/2. Quick question: imagine the same list for base 7 instead of base 2. Would you have to redo all that work, or could you already predict the answer? Type your guess.
   Look for: Student senses the rule might transfer without redoing the table, even if unsure why yet.

2. **Say:** Here's the key move: instead of using a number for the base, we use a letter, b, and require that b≠0. The rule b^m · b^n = b^(m+n) works for positive whole exponents no matter what number b is, it's just repeated multiplication. If we want that SAME rule to keep working when exponents are 0 or negative, the values of b^0 and b^-n get forced. No base is special.
   Look for: Student can restate that using a letter instead of a number is what makes the argument apply to every base.

3. **Say:** Watch: b^1 · b^0 must equal b^1 if the addition rule holds. So b · b^0 = b. Divide both sides by b. What do you get for b^0? Does your answer depend on what b actually is?
   Look for: Student divides both sides by b, gets b^0=1, and notices b never had to be a specific number.

4. **Say:** Your turn, with a NEW base: use c (c≠0) instead of b. Start from c^n · c^-n = c^0. You know c^0=1 from the last step. Solve for c^-n. Show each step.
   Look for: Student writes c^n · c^-n = 1, divides by c^n, gets c^-n = 1/c^n, without needing a specific number for c.

5. **Say:** Notice what just happened: you proved it for c, a totally arbitrary base, using nothing but the addition rule and 'divide both sides.' That argument works word-for-word for 2, for 7, for -5, for any nonzero number. That's why the zero and negative exponent rules apply to every base, the proof never picked one.
   Look for: Student connects 'the letter could be replaced by any nonzero number' to 'the rule holds for every base.'

6. **Say:** Now you'll answer questions that ask you to build this same kind of any-base argument yourself, not just repeat what we did with 2. Watch for that difference in the questions ahead.
   Look for: Student understands upcoming items require generating the general argument, not recalling the base-2 version.

If they get stuck

- *Multiplying always makes a number bigger and dividing always makes it smaller, so a bigger exponent must give a bigger number.*
  Use a table extending 2^3, 2^2, 2^1, 2^0, 2^-1, 2^-2 side by side and have students describe the pattern in the values, not just the exponents, before naming a rule.
- *In scientific notation, moving the decimal point and changing the exponent are two separate, unrelated steps to memorize.*
  Anchor every conversion to a number line or place-value chart showing the decimal actually sliding across powers-of-ten columns, not a rule about counting digits.
- *Anything to the zero power should equal zero, because zero times anything is zero.*
  Extend the division pattern 2^3/2^3 = 2^0 = 1 explicitly and require students to state the pattern rule in their own words before naming it as a definition.

Read together FOR THE STUDENT

You already saw that 2^3, 2^2, 2^1 keep dividing by 2 as the exponent drops by 1. That pattern forces 2^0=1, 2^-1=1/2, 2^-2=1/4. Here's the big question: does this only work for 2? Try it with 5, or with any letter b. Every time you drop the exponent by 1, you divide by the base. That division rule never mentions which base you picked, so it has to work for every nonzero base, not just 2.

Going further: For base b, the pattern b^n / b = b^(n-1) holds for positive integer exponents because it's just repeated multiplication. Extending the exponents to 0 and negatives means DEFINING b^0 and b^-n so this same division pattern keeps working with no exceptions. Since the argument uses only b^n / b = b^(n-1), never a specific number, it applies to any nonzero base b. That's why b^0=1 and b^-n=1/b^n for every nonzero b, not just the base 2 used in the examples above.

Watch it work

Show that b^0 = 1 must be true for ANY nonzero base b, not just b=2.

1. Start from the rule that already works for positive exponents: b^m · b^n = b^(m+n), true for whole numbers m,n because it's just repeated multiplication.
2. Pick m = n = 1, but keep b as a letter, not a number: b^1 · b^0 would need to equal b^(1+0) = b^1, IF the rule keeps working.
3. So b · b^0 = b. Divide both sides by b (allowed since b ≠ 0): b^0 = 1.
4. Nothing in that division used which number b was, it works the same for b=2, b=7, or b=193. So b^0=1 for every nonzero base.

The idea: The rule b^0=1 isn't observed case by case, it's forced by requiring the addition rule to keep working, and that force applies to every base at once.

Show that b^-n = 1/b^n must be true for ANY nonzero base b.

1. Use the same rule b^m · b^n = b^(m+n), and pick m = n and the other exponent as -n: b^n · b^-n should equal b^(n + -n) = b^0.
2. We already showed b^0 = 1 for any nonzero b. So b^n · b^-n = 1.
3. Divide both sides by b^n (allowed since b ≠ 0, so b^n ≠ 0): b^-n = 1/b^n.
4. Every step used the general letter b, never a specific number, so the result holds for all nonzero bases at once.

The idea: Negative exponents are defined, not discovered, and the definition falls out of demanding the addition rule survive, for every base, simultaneously.

Clareza Grade 8 · Math Lesson 8 Name: _____

Practice: Why zero and negative exponents always work

1. Which statement about 5^0 is correct?

(A) 5^0 = 0, because zero times anything is zero

(B) 5^0 is undefined because there's nothing to multiply

(C) 5^0 = 1, forced by keeping the pattern of dividing by 5 each time the exponent drops by 1

2. A student says '5 > 2, so (1/2)^5 > (1/2)^2.' What is the flaw?

(A) Nothing is wrong; (1/2)^5 really is bigger than (1/2)^2

(B) They assumed a bigger exponent always gives a bigger result, which fails once the base is between 0 and 1

(C) They made an arithmetic mistake multiplying fractions

3. Someone claims b^0 = 1 only for whole-number bases like 2, 5, or 10, not for something like b = 3/4. Using the argument from the lesson, is this claim correct?

(A) Correct, fractions behave differently, so the proof would need to be redone for b = 3/4

(B) Incorrect, but only because 3/4 is close enough to 1 that it happens to still work

(C) Incorrect, the proof only required b≠0, so it applies to b=3/4 exactly the same way

4. Which fact was ESSENTIAL to the proof that b^-n = 1/b^n works for any base, in the lesson's argument?

(A) That b is not zero, so dividing by b^n is allowed (B) That n is an even number

(C) That b is a positive whole number and not a fraction or a decimal

5. In 4,500,000 = 4.5 x 10^6, why is the exponent 6 rather than some other number?

(A) Because the exponent is chosen to match the number of digits before the decimal point

(B) Because the decimal point was moved 6 places, and each place-move corresponds to one factor of 10

(C) Because 4,500,000 has 6 zeros written at the end, and you count zeros

6. A classmate writes this 'proof' for base 2 only: '2^1·2^0=2^1, so 2·2^0=2, so 2^0=1.' They say this proves b^0=1 for every base. What's missing from calling it a general proof?

- **A** The argument needs to be checked and confirmed for at least 5 different number bases before it can count as a general proof for every base
- **B** The argument needs to be redone with a letter (any nonzero b) instead of the number 2, so no specific base is assumed
- **C** Nothing is missing at all; plugging in the one number 2 is enough to prove it, since all bases behave in exactly the same way as 2 does

7. Look at the pattern: 10^3 = 1000, 10^2 = 100, 10^1 = 10. Each time the exponent drops by 1, you divide by 10. Following the pattern, what is 10^0?

- **A** Undefined, because the pattern covers only positive exponents
- **B** 1, because 10 divided by 10 is 1
- **C** 0, because zero times anything is zero

8. Write 6.5 x 10^3 as an ordinary number.

- **A** 650, moving the decimal 2 places
- **B** 65,000, moving the decimal 4 places
- **C** 6,500, moving the decimal 3 places

9. Which is larger, 10^-2 or 10^-5?

- **A** Neither, because both are 0 for negative exponents
- **B** 10^-2, because 1/100 is larger than 1/100,000
- **C** 10^-5, because 5 is bigger than 2

10. In 0.00072 = 7.2 x 10^-4, why is the exponent -4 rather than some other number?

- **A** The decimal moves 4 places, and each place is one more factor of 1/10
- **B** The decimal move and the exponent are separate steps, so -4 is just memorized
- **C** Multiplying always makes things bigger, so the minus sign is only decoration on a 4

11. A student argues: 'b^1 = b, and dropping the exponent by 1 divides by b, so b^0 = b ÷ b = 1.' Another student asks, 'What if b = 0?' Which response is correct?

- **A** The step divides by b, so b must not be 0, but every nonzero b works
- **B** Zero times anything is zero, so b^0 should be 0 for every base, even this one
- **C** The argument covers only the numbers tried, since a letter b cannot be divided

12. Using base 1/2: (1/2)^2 = 1/4 and (1/2)^1 = 1/2. Dropping the exponent by 1 means dividing by the base 1/2. What is (1/2)^-1, and why?

- **A** 1/4, because dividing always makes it smaller and a lower exponent gives a lower number
- **B** 0, because zero times anything is zero once the exponent goes below zero
- **C** 2, because 1 ÷ 1/2 = 2, and dividing by a fraction less than 1 makes a bigger number

Finding mistakes in exponent solutions



Goal: Critique a flawed worked example that misapplies an exponent rule (e.g. adding exponents with different bases, or mishandling a negative exponent's sign) and identify precisely where the reasoning fails.

About 28 minutes

Teach it FOR THE PARENT

1. **Say:** Quick retrieval: what is 2^0? What is 2^-3? Type your two answers before we start.
   Look for: 2^0 = 1 and 2^-3 = 1/8. Flags zero/negative misconceptions early.
2. **Say:** Every exponent rule has a condition. Same-base rule needs matching bases. Negative exponent rule means 'flip to a fraction', not 'make it negative'. Today you are the checker, not the solver. You will find where someone else's steps broke a rule's condition.
   Look for: Student can restate that rules only apply under specific conditions.
3. **Say:** Watch this: '3^2 x 4^3 = 12^5.' Step 1, I check the bases. Step 2, they don't match, so I can't add exponents. Step 3, I compute both sides separately to see the real values. Where exactly did the mistake happen?
   Look for: Student names the base-mismatch step, not just 'the answer is wrong'.
4. **Say:** Now this one: '5^-2 = -25.' Step 1, what does a negative exponent actually mean? Step 2, apply that rule correctly. Step 3, compare to the flawed claim. What went wrong?
   Look for: Student identifies that the sign got applied to the answer instead of triggering a reciprocal.
5. **Say:** Your turn. Someone wrote: '4,500,000 = 4.5 x 10^3.' Find the exact step that is wrong, and say which rule or idea it breaks.
   Look for: Student states the exponent should be 6, tied to how many places the decimal moved.
6. **Say:** You just did the real job: not solving, but checking someone else's work step by step. The practice set gives you more flawed examples. Find the exact broken step every time, not just the wrong final answer.
   Look for: Student is ready to critique flawed steps in the practice items, not just recompute answers.

If they get stuck

- *Multiplying always makes a number bigger and dividing always makes it smaller, so a bigger exponent must give a bigger number.*
  Use a table extending 2^3, 2^2, 2^1, 2^0, 2^-1, 2^-2 side by side and have students describe the pattern in the values, not just the exponents, before naming a rule.
- *In scientific notation, moving the decimal point and changing the exponent are two separate, unrelated steps to memorize.*
  Anchor every conversion to a number line or place-value chart showing the decimal actually sliding across powers-of-ten columns, not a rule about counting digits.
- *Anything to the zero power should equal zero, because zero times anything is zero.*
  Extend the division pattern 2^3/2^3 = 2^0 = 1 explicitly and require students to state the pattern rule in their own words before naming it as a definition.

Read together FOR THE STUDENT

Exponent rules are like traffic rules. Each one only works when its condition is met. You can add exponents only when the bases match: 2^3 x 2^4 = 2^7. You cannot add exponents when bases differ, like 2^3 x 3^4. A negative exponent does not make a number negative. It means 'flip to a fraction': 2^-3 = 1/8, a small positive number. When you critique a worked example, find the exact line where a rule got used outside its condition.

Going further: Every exponent law has a condition attached. The product rule a^m x a^n = a^(m+n) requires the same base a. The negative exponent rule a^-n = 1/a^n changes size, not sign, the result stays positive when a is positive. Zero as an exponent means 'no repeated factor', giving 1, not 0. To critique a flawed example, do not just say 'wrong.' Point to the specific step, name the rule that was misapplied, and state the condition it broke.

Watch it work

Find the error: 3^2 x 4^3 = 12^5. Explain what went wrong.

1. Check the bases: 3^2 has base 3. 4^3 has base 4.
2. The product rule a^m x a^n = a^(m+n) needs the SAME base. Bases differ here.
3. So exponents cannot add. The step '3^2 x 4^3 = 12^5' breaks the same-base condition.
4. Correct value: 3^2 = 9, 4^3 = 64. 9 x 64 = 576, not 12^5 (248,832).

The idea: The product-of-powers rule only fires when both bases are identical.

Find the error: 5^-2 = -25. Explain what went wrong.

1. A negative exponent means 'reciprocal', not 'negative number'.
2. Rule: a^-n = 1/a^n. So 5^-2 = 1/5^2.
3. 1/5^2 = 1/25, a small positive fraction.
4. The error treated the minus sign on the exponent as a minus sign on the answer. Correct answer: 1/25.

The idea: A negative exponent shrinks the number toward zero; it never flips its sign.

 Name: _____

Practice: Finding mistakes in exponent solutions

1. A student writes: 2^3 x 2^4 = 4^7. Where is the error?

(A) 2^7 should equal 4^7 because 2 x 2 = 4.

(B) The exponents should be multiplied, not added.

(C) The base should stay 2, not become 4.

(D) Exponents should not be added at all here.

2. A student writes: 6^0 = 0. What is the flaw in this reasoning?

(A) 6^0 is undefined, so no value is correct.

(B) They should have written 6^0 = 6.

(C) The exponent 0 only ever works for even bases, and 6 happens to be even, so 0 is right.

(D) They treated the exponent 0 as meaning 'zero times the base'.

3. A student claims: 3^-4 is bigger than 3^2, because 'the exponent looks bigger in absolute value.' Where does this reasoning break?

(A) A negative exponent gives a small fraction, not a large number.

(B) 3^-4 should actually be negative, so it can't be bigger.

(C) You cannot compare exponents with different signs at all.

(D) 3^2 should be rewritten with a negative exponent to compare fairly.

4. A worked solution says: '(1/2)^5 is bigger than (1/2)^2 because 5 is bigger than 2.' Find the flawed step.

(A) (1/2)^5 and (1/2)^2 cannot be compared since bases match.

(B) The comparison ignored that the base is a fraction less than 1.

(C) The fraction 1/2 should be converted to a decimal before any exponent rule applies.

(D) The exponents should have been added first.

5. A student converts: '450,000 = 4.5 x 10^4.' Find where the reasoning breaks.

- **A** The decimal moved 5 places, so the exponent should be 5, not 4.
- **B** The exponent should be negative since the original number has zeros.
- **C** Scientific notation cannot be used for numbers ending in zeros.
- **D** The number 4.5 is wrong; it should be 45.

6. A student converts: '0.0032 = 3.2 x 10^3.' Where exactly does this go wrong?

- **A** The exponent's sign is wrong; it should be -3, not 3.
- **B** 3.2 should be 0.32 to keep the value the same.
- **C** 0.0032 cannot be written in scientific notation.
- **D** The exponent should be 4, matching the number of decimal places shown.

7. A worked solution: '2^5 / 2^8 = 2^3, since 8 - 5 = 3.' Find the flawed step.

- **A** 2^5 / 2^8 cannot be simplified using any rule.
- **B** Division of powers requires different bases, not the same base.
- **C** The subtraction order is backward; it should be 5 - 8 = -3.
- **D** The answer should be 2^13 because exponents add during division.

8. A student writes: 5^2 x 7^3 = 5^5, because 2 + 3 = 5. Where is the error?

- **A** Exponents add only when the bases match, and 5 and 7 differ.
- **B** The base should become 12 after adding, giving 12^5.
- **C** Multiplying makes bigger numbers, so the exponents should be multiplied.
- **D** Nothing is wrong, because 2 + 3 = 5 is correct arithmetic.

9. A student writes: 4^-2 = -16. What is the flaw in this reasoning?

- **A** A negative exponent means flip to a fraction, so 4^-2 = 1/16, not negative.
- **B** The sign should be ignored, so 4^-2 is the same as 4^2 = 16.
- **C** Anything to a negative power equals zero, so 4^-2 = 0.
- **D** The exponent multiplies the base, so 4 x -2 = -8 is the answer.

10. A student reasons: 'Dividing always makes a number smaller, so 8 / 2^-1 must be less than 8.' Where does this reasoning break?

- **A** 2^-1 = 1/2, and dividing by 1/2 gives 16, which is bigger than 8.
- **B** 2^-1 = -2, so 8 / 2^-1 = -4, which is less than 8.
- **C** The reasoning is fine, since dividing by any power of 2 shrinks 8.
- **D** 2^-1 = 1/2, and 8 / (1/2) = 4, which is smaller than 8.

11. A student converts: '83,000,000 = 8.3 x 10^6, because there are six zeros.' Where does the reasoning break?

- **A** The exponent should be 8, since 83,000,000 has eight digits.
- **B** Moving the decimal and choosing the exponent are separate steps, so any exponent works.
- **C** The exponent should be negative, because 8.3 is smaller than 83,000,000.
- **D** The exponent counts the decimal places moved, which is 7, not the zeros.

12. A student simplifies (2^3 x 2^-5) / 2^-1 and writes: Step 1: 2^3 x 2^-5 = 2^-2. Step 2: 2^-2 = -4. Step 3: -4 / 2^-1 = -4 / (-2) = 2. Which is the first step that goes wrong?

- **A** Step 2, because a negative exponent flips to a fraction: 2^-2 = 1/4, not -4.
- **B** Step 3, because dividing always makes a number smaller, and 2 is bigger than -4.
- **C** No step is wrong, because a negative exponent makes the result negative.
- **D** Step 1, because 2^3 and 2^-5 have different exponents, so they cannot be combined.

13. A student simplifies (4^3 x 4^2) / 2^5 and writes: Line 1: 4^3 x 4^2 = 4^5. Line 2: 4^5 / 2^5 = 2^(5 - 5) = 2^0. Line 3: 2^0 = 0. Which is the first line where a rule is used outside its condition?

- **A** Line 3, because 2^0 should be 1, not 0.
- **B** Line 1, because the exponents 3 and 2 are different, so they cannot be added.
- **C** None, because each line follows a valid exponent rule from the lesson.
- **D** Line 2, because subtracting exponents needs matching bases, and 4 and 2 differ.

Answer key

Math

Lesson 1	1. B 2. C 3. C 4. B 5. B 6. A 7. C 8. B 9. B 10. C 11. C 12. A 13. A
Lesson 2	1. A 2. B 3. B 4. A 5. B 6. C 7. B 8. B 9. A 10. C 11. B 12. A 13. A 14. B
Lesson 3	1. B 2. B 3. A 4. C 5. A 6. B 7. C 8. B 9. A 10. B 11. B 12. C
Lesson 4	1. B 2. B 3. B 4. A 5. B 6. C 7. C 8. C 9. C 10. B 11. C 12. C 13. B 14. B
Lesson 5	1. C 2. B 3. A 4. B 5. A 6. C 7. C 8. D 9. C 10. C 11. C 12. A 13. B 14. C
Lesson 6	1. C 2. B 3. A 4. B 5. A 6. C 7. A 8. A 9. B 10. A 11. B 12. C 13. C 14. C
Lesson 7	1. A 2. A 3. A 4. B 5. B 6. A 7. C 8. A 9. C 10. B 11. C 12. A
Lesson 8	1. C 2. B 3. C 4. A 5. B 6. B 7. B 8. C 9. B 10. A 11. A 12. C
Lesson 9	1. C 2. D 3. A 4. B 5. A 6. A 7. C 8. A 9. A 10. A 11. D 12. A 13. D
Unit 1 check-up	1. C 2. B 3. B 4. B 5. C 6. C 7. C 8. C 9. B 10. B 11. B 12. A 13. C 14. A 15. B 16. B 17. A 18. A

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