CLAREZA ACADEMY

Grade 7 -- Mathematics Math

Free sample: Unit 1, Lessons 1 to 9

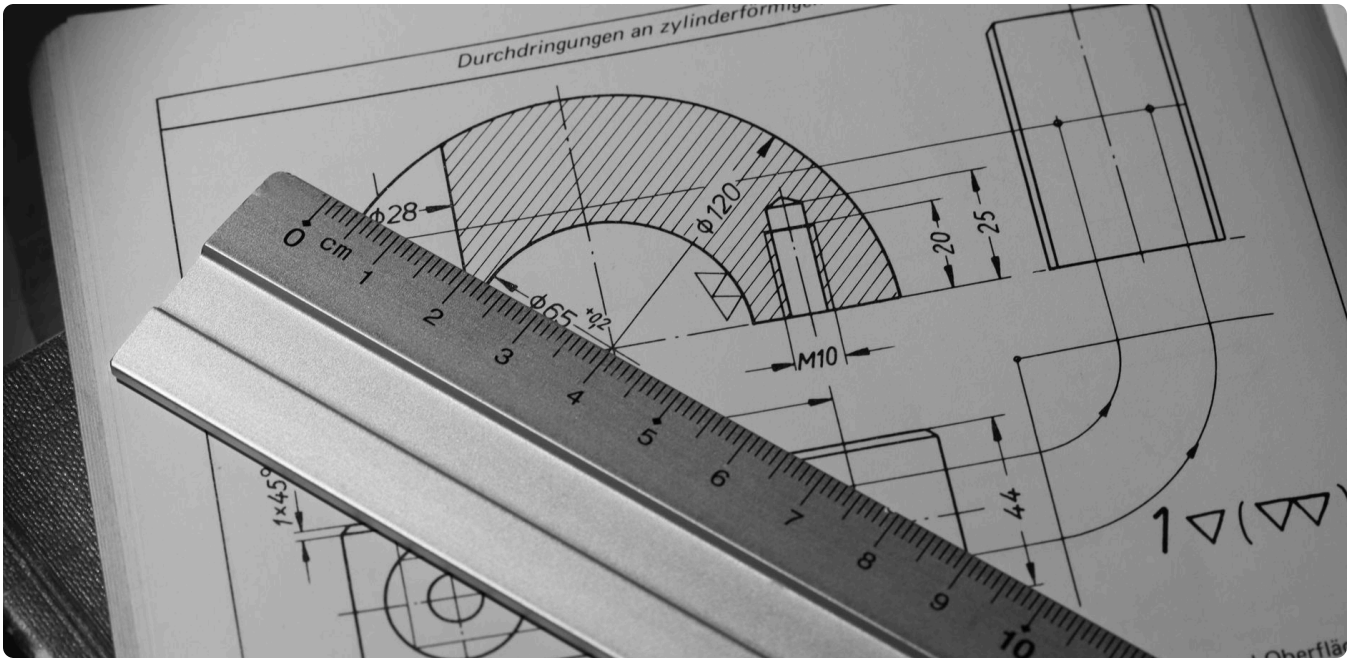
These are 9 lessons from *Clareza Grade 7: Mathematics*, a seventh grade year a parent runs at the kitchen table. Each lesson is a short script for you, a page to read together, and a practice sheet you may copy for your household. Answers are at the back.

Clareza Grade 7 : Mathematics. Sample edition.

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Clareza is a secular curriculum. Standards references: Common Core State Standards and the Next Generation Science Standards.



MATH · UNIT 1

Proportional Relationships

This unit is about one question asked five different ways: is the ratio between two quantities always the same? Your kid learns to check that using fraction unit rates, tables, graphs, and equations, then applies it to real percent problems like tax, tip, and discount.

In this unit

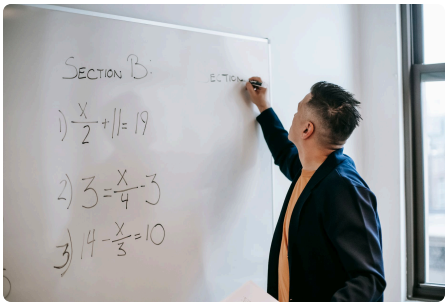
1. Unit rates with fractions
2. Constant of proportionality from tables
3. Deciding if a relationship is proportional
4. Straight lines that miss the origin
5. Writing the equation y = kx
6. What the point (1, r) tells us
7. Tax, tip, markup, and discount problems
8. Does percent change order matter
9. Proportions with a fixed fee

MATH · UNIT 1 · LESSON 1

Unit rates with fractions

Goal: Given a rate involving fractional quantities (e.g. 3/4 cup per 2/3 hour), compute the unit rate.

About 25 minutes



Teach it FOR THE PARENT

1. **Say:** Quick retrieval: last unit, a car went 60 miles in 2 hours. What's the unit rate? Answer before you scroll.
   Look for: Student answers 30 miles per hour, recalling whole-number unit rate = total ÷ time.
2. **Say:** Today the numbers are fractions. A unit rate is still 'amount per ONE unit.' We still divide. The only new part: dividing fractions means flip the second one and multiply.
   Look for: Student can state that unit rate = first quantity divided by second quantity.
3. **Say:** Watch this one: 3/4 cup of water every 2/3 hour. I write 3/4 ÷ 2/3. I flip 2/3 to 3/2. I multiply: 3/4 × 3/2 = 9/8. That simplifies to 1 1/8 cups per hour. What did I do first?
   Look for: Student identifies: wrote the division, then flipped the second fraction before multiplying.
4. **Say:** Here's the trap: 9/8 is BIGGER than 3/4. Multiplying by a fraction less than 1 made a bigger answer. Why did that happen?
   Look for: Student explains dividing by a number smaller than 1 increases the result, unlike dividing by a whole number.
5. **Say:** Your turn, fresh numbers: a painter covers 5/6 of a wall in 3/4 hour. Set up the division, flip, multiply, and simplify. Show each step.
   Look for: Student writes 5/6 ÷ 3/4, flips to 4/3, multiplies to 20/18, simplifies to 10/9.
6. **Say:** You just turned a fraction rate into a unit rate the same way every time: divide, flip the second fraction, multiply, simplify. The practice set below uses that exact routine.
   Look for: Student is ready to apply the same four-step routine independently.

If they get stuck

- *If a graph is a straight line, the relationship must be proportional.*
  Show two straight lines side by side, one through the origin and one not. Ask students to test both with the same table-ratio check (y/x constant) before deciding.
- *In a proportional table, if you just check whether y increases when x increases, that is enough to call it proportional.*
  Require students to compute y/x for every row, not just the first, and to state the ratio out loud before deciding.
- *Multiplying by a fraction less than 1 (like the unit rate 3/4) should make the answer bigger, so something is wrong.*
  Use a double number line to show the multiplication landing between two whole-number benchmarks, making the smaller result visibly reasonable.

Read together FOR THE STUDENT

A unit rate tells you how much of something happens in ONE unit of something else, like 1 hour. If a recipe uses 3/4 cup of flour every 2/3 hour, divide 3/4 by 2/3 to find cups per hour. Dividing fractions means flip the second fraction and multiply. Think of splitting a pizza: dividing by a small fraction gives more pieces, not fewer. So dividing by a fraction less than 1 can make your answer bigger than you expect. That is normal, not a mistake.

Going further: A unit rate is the amount per ONE unit, written as a/b where b = 1. With fractional quantities, divide the first fraction by the second: (rate quantity) ÷ (time or other unit). To divide fractions, multiply by the reciprocal: a/b ÷ c/d = a/b × d/c. Example: 3/4 cup ÷ 2/3 hour = 3/4 × 3/2 = 9/8 cups per hour. Dividing by a fraction less than 1 gives a bigger result than the original number, that is expected, not an error.

Watch it work

A runner drinks 3/4 cup of water every 2/3 hour. Find the unit rate in cups per hour.

1. Write the rate as a division: 3/4 cup ÷ 2/3 hour.
2. Flip the second fraction to get its reciprocal: 2/3 becomes 3/2.
3. Multiply instead of dividing: 3/4 × 3/2 = 9/8.
4. Simplify: 9/8 = 1 1/8 cups per hour.

The idea: A unit rate always means dividing the first quantity by the second quantity, even when both are fractions.

A painter covers 5/6 of a wall in 3/4 hour. Find the unit rate in walls per hour.

1. Write the division: 5/6 wall ÷ 3/4 hour.
2. Take the reciprocal of 3/4: it becomes 4/3.
3. Multiply: 5/6 × 4/3 = 20/18.
4. Simplify by dividing top and bottom by 2: 20/18 = 10/9 walls per hour.

The idea: Simplify the final fraction fully, a reduced fraction or mixed number is the finished unit rate.

 Name: _____

Practice: Unit rates with fractions

1. A hose fills 1/2 gallon every 1/4 hour. Find the unit rate in gallons per hour.

(A) 1/2 gallons per hour (B) 2 gallons per hour (C) 1/8 gallons per hour

2. A car uses 3/5 gallon of gas every 1/2 hour. Find the unit rate in gallons per hour.

(A) 5/6 gallons per hour (B) 6/5 gallons per hour (C) 3/10 gallons per hour

3. A runner drinks 3/4 cup of water every 2/3 hour. Find the unit rate in cups per hour.

(A) 8/9 cup per hour (B) 1/2 cup per hour (C) 1 1/8 cups per hour

4. A student says: 'Dividing 3/4 by 2/3 gave 9/8, which is bigger than 3/4. That can't be right.' What is wrong with this thinking?

(A) The student is right about this. The answer to 3/4 divided by 2/3 should come out smaller than 3/4.

(B) Nothing is wrong. Dividing by a fraction less than 1 gives a larger result.

(C) The mistake the student made is using fractions instead of decimals, which would have given a smaller answer.

5. A painter covers 5/6 of a wall in 3/4 hour. Find the unit rate in walls per hour.

(A) 9/10 wall per hour (B) 5/8 wall per hour (C) 10/9 walls per hour

6. A recipe uses 2/3 cup of sugar for every 3/8 batch of cookies. Find the unit rate in cups per batch.

(A) 16/9 cups per batch (B) 9/16 cup per batch (C) 1/4 cup per batch

7. A table shows time and distance: (1/2, 1), (1, 2), (3/2, 3). A student says this is proportional because the numbers go up together. Is that enough to prove it is proportional?

(A) No. Tables can never show proportional relationships at all, because only a graph can show whether a line goes through the origin.

(B) Yes. If both of the columns in the table increase together, then the table shows a proportional relationship.

(C) No. You must check that distance divided by time is the same constant ratio each time.

8. A factory produces 7/8 ton of steel every 5/6 hour. Find the unit rate in tons per hour.

(A) 21/20 tons per hour (B) 20/21 ton per hour (C) 35/48 ton per hour

9. A machine fills 3/4 liter of juice every 1/2 minute. Find the unit rate in liters per minute.

(A) 3/8 liter per minute (B) 3/2 liters per minute (C) 2/3 liter per minute

10. A cyclist rides 4/5 mile in 1/3 hour. Find the unit rate in miles per hour.

(A) 4/15 mile per hour (B) 5/12 mile per hour (C) 12/5 miles per hour

11. A baker uses 5/6 pound of butter for every 2/3 of a batch. Find the unit rate in pounds per batch.

(A) 4/5 pound per batch (B) 5/4 pounds per batch (C) 5/9 pound per batch

12. A straight-line graph passes through (1/2, 3) and (1, 4) and crosses the y-axis at 2. A student says: 'It is a straight line, so it is proportional.' Is the student right?

(A) No, it misses (0, 0) and y ÷ x changes from 6 to 4.

(B) Yes, any straight line is a proportional relationship.

(C) Yes, because y gets larger as x gets larger.

13. A hiker covers 5/6 mile in 7/9 hour. Find the unit rate in miles per hour.

(A) 14/15 mile per hour (B) 15/14 miles per hour (C) 35/54 mile per hour

14. A student reads 5/6 of a chapter in 3/10 hour. Find the unit rate in hours per chapter.

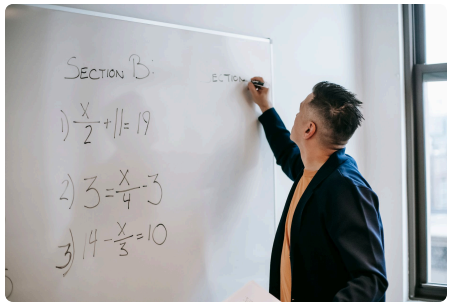
(A) 25/9 hours per chapter (B) 9/25 hour per chapter (C) 1/4 hour per chapter

MATH · UNIT 1 · LESSON 2

Constant of proportionality from tables

Goal: Given a table of paired values, determine the constant of proportionality k by computing y/x for each row.

About 25 minutes



Teach it FOR THE PARENT

1. **Say:** A juice recipe uses 2 cups of water for every 1 cup of mix. If I gave you a table of cups, could you find the hidden number that connects them every time?
   Look for: Student shows curiosity about a fixed ratio; no answer needed yet.
2. **Say:** That hidden number is called k, the constant of proportionality. You find it by dividing y by x in any row. If the table is truly proportional, you get the same k every time.
   Look for: Student can restate: divide y by x to find k.
3. **Say:** Let's try x: 2, 4, 6 and y: 8, 16, 24. Divide y by x in each row. What do you notice about the three answers?
   Look for: Student notices all three divisions equal 4, confirming k = 4.
4. **Say:** Quick check: in a table where x: 3, 6 and y: 12, 24, what is k?
   Look for: Student divides 12 ÷ 3 and 24 ÷ 6, both equal 4, so k = 4.
5. **Say:** New table: x: 4, 8, 12 and y: 3, 6, 9. Find k. Divide y by x in each row and write down what you get.
   Look for: Student computes 0.75 for all three rows and states k = 0.75.
6. **Say:** Watch out: a straight line on a graph is not automatically proportional. And 'y goes up when x goes up' is not enough either. Only a constant y/x proves it.
   Look for: Student can say why increasing together does not prove proportionality.
7. **Say:** Now you'll practice finding k from tables on your own. Divide y by x in every row, every time.
   Look for: Student begins practice items applying the row-by-row division method.

If they get stuck

- *If a graph is a straight line, the relationship must be proportional.*
  Show two straight lines side by side, one through the origin and one not. Ask students to test both with the same table-ratio check (y/x constant) before deciding.
- *In a proportional table, if you just check whether y increases when x increases, that is enough to call it proportional.*
  Require students to compute y/x for every row, not just the first, and to state the ratio out loud before deciding.
- *Multiplying by a fraction less than 1 (like the unit rate 3/4) should make the answer bigger, so something is wrong.*
  Use a double number line to show the multiplication landing between two whole-number benchmarks, making the smaller result visibly reasonable.

Read together FOR THE STUDENT

A table can hide a hidden rule that connects x and y. To find it, divide y by x in each row. If you always get the same number, that number is called the constant of proportionality, k. Think of a lemonade stand: if 2 cups need 6 lemons, and 3 cups need 9 lemons, dividing y by x always gives 3. That 3 is k, the fixed rate that connects cups and lemons every time.

Going further: The constant of proportionality, k, is the fixed ratio between y and x in a proportional table. Find it by computing y/x for each row. If the table is truly proportional, every row gives the same k. This works whether k is a whole number, a decimal, or a fraction. Once you have k, you can write the relationship as y = kx and use it to find any missing value.

Watch it work

Find k from this table: x: 2, 4, 6 y: 8, 16, 24

1. Divide y by x in row 1: 8 ÷ 2 = 4.
2. Divide y by x in row 2: 16 ÷ 4 = 4.
3. Divide y by x in row 3: 24 ÷ 6 = 4.
4. Every row gives 4, so k = 4.

The idea: When a table is proportional, y/x is the same value in every row, that value is k.

Find k from this table: x: 4, 8, 12 y: 3, 6, 9

1. Divide y by x in row 1: 3 ÷ 4 = 0.75.
2. Divide y by x in row 2: 6 ÷ 8 = 0.75.
3. Divide y by x in row 3: 9 ÷ 12 = 0.75.
4. Every row gives 0.75, so k = 0.75.

The idea: k can be a decimal less than 1, the same division rule still applies.

 Name: _____

Practice: Constant of proportionality from tables

1. x: 3, 6, 9 y: 12, 24, 36 What is k?

(A) 4 (B) 9 (C) 3

2. x: 5, 10, 15 y: 20, 40, 60 What is k?

(A) 4 (B) 5 (C) 15

3. x: 2, 5, 8 y: 6, 15, 24 What is k?

(A) 3 (B) 4 (C) 2

4. x: 1, 2, 3 y: 5, 8, 11 A student says this table is proportional because y increases as x increases. What is wrong with that thinking?

(A) Nothing, increasing together always means proportional.

(B) The table needs more rows to decide.

(C) y/x is not the same in every row, so it is not proportional.

5. x: 4, 6, 10 y: 3, 4.5, 7.5 What is k?

(A) 0.75 (B) 4 (C) 1.25

6. x: 8, 12, 20 y: 6, 9, 15 A student divides 6÷8 and gets 0.75. They think this must be wrong because dividing made the number smaller. Are they right to worry?

(A) No, k can be a decimal less than 1 and still be correct.

(B) Yes, dividing should always make numbers smaller so this seems off.

(C) No, but only because the x values are too large.

7. x: 1/2, 1, 3/2 y: 2, 4, 6 What is k?

(A) 0.5 (B) 2 (C) 4

8. A graph shows a straight line through the points (1,4), (2,7), (3,10). A student says this must be proportional because the graph is a straight line. Use the table values to check.

(A) They are right, but only because the numbers increase evenly.

(B) They are right, any straight line is proportional.

(C) They are wrong, y/x is not constant across the points.

9. x: 2, 4, 6 y: 10, 20, 30 What is k?

(A) 5 (B) 8 (C) 1/5

10. A table shows hours worked (x) and pay in dollars (y). x: 2, 4, 6 y: 18, 36, 54 What is k?

(A) 9 (B) 16 (C) 18

11. Which table shows a proportional relationship between x and y?

(A) x: 2, 4, 6 and y: 6, 12, 18 (B) x: 2, 4, 6 and y: 6, 10, 14 (C) x: 2, 4, 6 and y: 8, 12, 16

12. x: 5, 10, 25 y: 2, 4, 10 What is k?

(A) 5/2 (B) 2/5 (C) 3

13. A graph shows a straight line through the points (2, 5), (4, 9), and (6, 13). A student says the relationship must be proportional because the graph is a straight line. Is the student correct?

(A) Yes, because every straight line has a constant ratio y/x

(B) No, because y/x is 2.5, then 2.25, then about 2.17

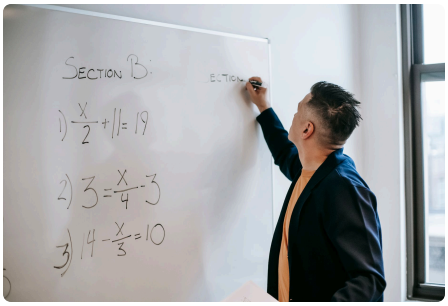
(C) Yes, because y goes up by 4 each time x goes up by 2

14. This table is proportional. x: 15, 20, 35 y: 6, 8? What is the missing y value?

(A) 14 (B) 23 (C) 87.5

MATH · UNIT 1 · LESSON 3

Deciding if a relationship is proportional



Goal: Given two representations of the same context (a table and a graph, or a table and a verbal description) that are NOT both provided directly, determine whether the underlying relationship is proportional by checking whether the ratio y/x is constant across all given pairs, not just the first pair or the visual shape of the graph.

About 28 minutes

Teach it FOR THE PARENT

1. **Say:** Imagine a graph of a gym membership that is a perfectly straight line. Two points on it are (2 months, $70) and (4 months, $120). Is cost proportional to months? Don't answer yet. Just notice that a straight line feels like it should be proportional.
   Look for: Student notices the line is straight and guesses proportional, or hesitates because of a possible fee.
2. **Say:** Straight does not always mean proportional. A relationship is proportional only if y divided by x gives the SAME number for every pair. Let's test the gym example: 70/2 = 35, but 120/4 = 30. Different answers, so it is NOT proportional, even though it's a straight line.
   Look for: Student states or restates: check y/x at multiple points, not just the shape of the graph.
3. **Say:** Quick check: a graph line does NOT pass through (0,0). Can it still be proportional? Why or why not?
   Look for: Student says no, because proportional lines must pass through the origin.
4. **Say:** Now the flour and sugar example. The words say 3 cups flour for every 4 cups sugar. Imagine a table with the pairs (6, 8) and (9, 12). Step 1: turn the words into a ratio, 4/3. Step 2: check 8/6. Step 3: check 12/9. Step 4: compare all three ratios.
   Look for: Student computes 8/6 = 4/3 and 12/9 = 4/3, and concludes all ratios match.
5. **Say:** Your turn. Imagine a table for a phone plan with the pairs (2 GB, $10) and (5 GB, $25). A second plan is described as 'always $6 per GB.' Which plan, if any, is proportional? Find y/x for every pair before deciding.
   Look for: Student computes 10/2=5 and 25/5=5 for plan one (proportional), and recognizes plan two's constant rate of $6/GB also makes it proportional.
6. **Say:** From now on, whenever you see a table, a graph, or a description, do the same thing: turn it into pairs, compute y/x for each, and compare. Never trust a straight line or a first pair alone. Try the practice set now.
   Look for: Student is ready to apply y/x checking across mixed representations independently.

If they get stuck

- *If a graph is a straight line, the relationship must be proportional.*
  Show two straight lines side by side, one through the origin and one not. Ask students to test both with the same table-ratio check (y/x constant) before deciding.
- *In a proportional table, if you just check whether y increases when x increases, that is enough to call it proportional.*
  Require students to compute y/x for every row, not just the first, and to state the ratio out loud before deciding.
- *Multiplying by a fraction less than 1 (like the unit rate 3/4) should make the answer bigger, so something is wrong.*
  Use a double number line to show the multiplication landing between two whole-number benchmarks, making the smaller result visibly reasonable.

Read together FOR THE STUDENT

A relationship is proportional if y ÷ x gives the SAME number every single time. Think of buying apples at a steady price. If 2 apples cost $4 and 5 apples cost $10, both give $2 per apple. That is proportional. But if a taxi charges $3 just to start, plus $2 per mile, the ratio y/x changes every time. It is not proportional, even though the graph is a straight line. Always check y/x for every pair you are given, not just the first one, and not just whether the line looks straight.

Going further: A relationship is proportional only when y/x is constant for every pair in the data. Test at least two or three pairs, not just one. A straight-line graph is not proof by itself. A line can be straight but not proportional if it does not pass through the origin (0,0), like y = x + 3. When given a table, verbal description, or graph, convert what you have into pairs of (x, y) and compute y/x for each. If any ratio differs, the relationship is not proportional.

Watch it work

Imagine a graph of a gym membership. It is a straight line that passes through (2, 70) and (4, 120). Is cost proportional to months?

1. Read two points from the line: (2, 70) and (4, 120).
2. Compute y/x for each point: 70/2 = 35. Then 120/4 = 30.
3. 35 does not equal 30. The ratio is not constant.
4. The line is straight, but it is not proportional. It likely has a start-up fee.

The idea: A straight line only means proportional if y/x stays the same at every point, including checking it does not just look straight.

A recipe says: 'Use 3 cups of flour for every 4 cups of sugar.' A table for a bigger batch shows (flour=6, sugar=8) and (flour=9, sugar=12). Is sugar proportional to flour?

1. Turn the verbal rule into a ratio: sugar/flour = 4/3 for the original.
2. Check the table pairs. For (6, 8): 8/6 = 4/3.
3. For (9, 12): 12/9 = 4/3.
4. Every ratio equals 4/3. The relationship is proportional.

The idea: When a verbal description and a table describe the same context, convert both into y/x and compare across all pairs.

Practice: Deciding if a relationship is proportional

1. A table shows (x, y) pairs: (2, 6), (4, 12), (6, 18). Is this relationship proportional?

(A) No, because the numbers are different in each row.

(B) Yes, because y/x = 3 for every pair. (C) Yes, because y increases as x increases.

2. A table shows: (1, 2), (2, 5), (3, 10). Is this relationship proportional?

(A) No, because y/x is 2, then 2.5, then about 3.3, so it is not constant.

(B) Yes, because y always gets bigger when x gets bigger.

(C) Yes, because the first pair gives y/x = 2.

3. Imagine a graph that is a straight line passing through (0, 0) and (3, 9). Is this relationship proportional?

(A) Yes, because any straight line on a graph is proportional.

(B) No, because the two points are far apart from each other.

(C) Yes, because the line is straight and passes through the origin.

4. Imagine a graph with a straight line through (1, 5) and (3, 11). It does not pass through (0, 0). Is this relationship proportional?

(A) No, because the numbers 5 and 11 are both odd, and a proportional graph needs even values.

(B) No, because it does not pass through the origin and y/x is not constant.

(C) Yes, because the graph is a straight line, and every straight line is proportional.

5. A verbal description says: 'A parking garage charges a flat $5 fee plus $2 per hour.' A table lists (hours, cost): (1, 7), (3, 11). Is cost proportional to hours?

(A) Yes, because cost always increases as hours increase.

(B) No, because 7/1 = 7 and 11/3 ≈ 3.7, which are not equal.

(C) Yes, because the table pairs both fit the $2 per hour rate.

6. A recipe rule says: 'Mix 3/4 cup of oil for every 1 cup of flour.' A table shows (flour, oil): (4, 3), (8, 6). Is oil proportional to flour?

(A) No, because 3/4 cup seems too small to multiply correctly.

(B) No, because 3/4 is a fraction, and fractions can't be unit rates.

(C) Yes, because 3/4 divided into each pair gives the same ratio.

7. Imagine a graph with the points (2, 8), (5, 20), and (6, 23). Is distance proportional to time here?

(A) No, because 8/2=4, 20/5=4, but 23/6≈3.8, so it is not constant.

(B) Yes, because the points look roughly like a straight line.

(C) Yes, because most of the points give the same ratio.

8. A verbal rule says: 'Earnings are always 5/8 of hours worked, no starting bonus.' A table shows (hours, earnings): (8, 5), (16, 10). Is this proportional?

(A) No, because there is no bonus mentioned in the table.

(B) Yes, because 5/8 equals both 5/8 and 10/16 from the table.

(C) No, because 5/8 is less than 1, so earnings can't equal a fraction of hours.

9. A table shows (x, y) pairs: (1, 3), (2, 5), (3, 7). Is this relationship proportional?

(A) Yes, because the first pair gives a ratio of 3

(B) Yes, because y goes up when x goes up

(C) No, because y/x is 3, then 2.5, then 2.33

10. A table shows (tickets, dollars): (4, 8), (6, 12), (9, 18). Is the cost proportional to the number of tickets?

(A) No, because the tickets change from row to row

(B) Yes, because y/x is 2 for every pair

(C) No, because the cost rises by 4, then by 6

11. Imagine a graph that is a straight line through the points (2, 9) and (4, 13). Is this relationship proportional?

(A) Yes, because any straight line is proportional

(B) No, because 9/2 = 4.5 but 13/4 = 3.25

(C) Yes, because y gets bigger as x gets bigger

12. A verbal rule says: 'A snail crawls 2/3 of a meter every minute, with no head start.' A table shows (minutes, meters): (3, 2), (9, 6), (12, 8). Is distance proportional to time?

- **A** No, because the meters are smaller than the minutes
- **B** No, because the meters rise by uneven steps
- **C** Yes, because meters/minutes is 2/3 in every row

13. A verbal rule says: 'A runner covers 3/8 of a lap each minute, with no head start.' A table shows (minutes, laps): (8, 3), (12, 4.5), (20, 7.5). Is this proportional?

- **A** No, because laps are fewer than minutes, so it is off
- **B** Yes, because laps/minutes is 3/8 in every row
- **C** No, because 4.5 and 7.5 are not whole numbers

14. A verbal rule says: 'A savings jar starts with $4 and adds $2 each week.' A table shows (weeks, dollars): (2, 8), (4, 12), (6, 16). Is the amount proportional to the weeks?

- **A** Yes, because the first pair gives 8/2 = 4
- **B** Yes, because the amount rises by $4 each row
- **C** No, because the ratios are 4, 3, and 2.67

MATH · UNIT 1 · LESSON 4

Straight lines that miss the origin



Goal: Explain why a straight-line graph that does not pass through the origin represents a non-proportional relationship, using the constant-ratio definition rather than the visual shape alone.

About 25 minutes

Teach it FOR THE PARENT

1. **Say:** Imagine two graphs, and both are straight lines. One passes through (0,0). One crosses the y-axis at 3. Only one is proportional. Which one, and how do you know for sure, not just by looking?
   Look for: Student names the line through the origin and shows awareness that 'straight' alone isn't the test.
2. **Say:** Proportional means y divided by x gives the SAME number every time, for every point on the line. That includes the point (0,0), zero of anything costs zero. If a straight line starts anywhere else, its y/x ratio changes as you move along it. Straight tells you 'linear.' Constant ratio tells you 'proportional.'
   Look for: No response needed, sets up vocabulary: constant ratio, origin, linear vs proportional.
3. **Say:** Line through (0,0), (2,4), (5,10). Step 1: does it hit (0,0)? Yes. Step 2: 4/2 = 2. Step 3: 10/5 = 2. Same ratio both times, proportional. Follow the worked example above: check the origin first, then confirm with two ratios.
   Look for: Student can restate the two checks: origin point, and matching y/x ratios.
4. **Say:** Quick check: a line passes through (0,0) and (3,9). What is y/x? Is that enough by itself to call it proportional, or do you need a second point too?
   Look for: Student computes 9/3=3 and recognizes one ratio plus the origin point is the pattern, ideally checks a second point for certainty.
5. **Say:** New line: through (0,3), (2,7), (4,11). Does it pass through the origin? Find y/x at (2,7) and at (4,11). Are the ratios the same? Is this line proportional? Explain your reasoning using the ratios, not just the picture.
   Look for: Student states the line misses (0,0), computes 7/2=3.5 and 11/4=2.75, notes the ratios differ, and concludes non-proportional.
6. **Say:** Every question coming up gives you a graph, a table, or an equation. Your job every time: check the origin, check if y/x stays constant. Don't trust 'it looks straight', prove it with the ratio.
   Look for: Student is ready to apply the two-step check (origin + constant ratio) independently on new items.

If they get stuck

- *If a graph is a straight line, the relationship must be proportional.*
  Show two straight lines side by side, one through the origin and one not. Ask students to test both with the same table-ratio check (y/x constant) before deciding.
- *In a proportional table, if you just check whether y increases when x increases, that is enough to call it proportional.*
  Require students to compute y/x for every row, not just the first, and to state the ratio out loud before deciding.
- *Multiplying by a fraction less than 1 (like the unit rate 3/4) should make the answer bigger, so something is wrong.*
  Use a double number line to show the multiplication landing between two whole-number benchmarks, making the smaller result visibly reasonable.

Read together FOR THE STUDENT

A proportional graph is a straight line that goes through the point (0,0), the origin. Think of buying apples: 0 apples cost $0. If the line starts anywhere else, like at $5 before you buy anything, it's not proportional, even though it's still straight. Always check: does the line hit (0,0)? And does y divided by x stay the same number every time?

Going further: A relationship is proportional only if y/x equals the same constant k for every point, including (0,0). A straight line with equation $y = mx + b$ is proportional only when $b = 0$. If $b \neq 0$, the line is still linear (straight) but not proportional, because y/x changes as x changes, try two points and divide. The shape 'straight' tells you it's linear; only the constant ratio test tells you it's proportional.

Watch it work

Imagine a graph with a straight line through (0,0), (2,4), and (5,10). Is this proportional? Explain using ratios.

1. Check if the line passes through the origin: yes, (0,0) is on it.
2. Compute y/x for each point: $4/2 = 2$, and $10/5 = 2$.
3. Both ratios equal the same constant, $k = 2$, this matches the definition $y = kx$.
4. Since the ratio is constant and the line passes through (0,0), the relationship is proportional.

The idea: A constant y/x ratio, not just a straight shape, is what proportional means.

Imagine a graph with a straight line through (0,3), (2,7), and (4,11). Is this proportional? Explain using ratios.

1. Check the origin: at x=0, y=3, not 0, the line does not pass through (0,0).
2. Compute y/x anyway to test the ratio: 7/2 = 3.5, and 11/4 = 2.75.
3. The two ratios are different, so y/x is not constant.
4. The line is straight (linear), but because the ratio changes and it misses the origin, it is not proportional.

The idea: A straight line that misses the origin always has a non-constant y/x ratio, shape alone cannot prove proportionality.

Practice: Straight lines that miss the origin

1. A straight-line graph passes through (0,0) and (3,6). What is y/x at the point (3,6)?

(A) 9 (B) 18 (C) 3 (D) 2

2. Which point MUST be on the graph of any proportional relationship?

(A) (1,1) (B) (0,0) (C) (5,0) (D) (0,5)

3. A straight line crosses the y-axis at (0,4) instead of (0,0). What does this tell you?

(A) The relationship cannot be proportional.

(B) You need to check if y increases as x increases before deciding.

(C) The relationship is proportional because the line is still straight.

(D) It depends on whether x is positive or negative.

4. A table shows (1,2), (2,5), (3,10). A student says it's proportional because y goes up as x goes up. What's wrong with this reasoning?

(A) Increasing together doesn't mean the ratio y/x is constant, check the ratios.

(B) The reasoning is wrong only because the x-values in the table aren't evenly spaced out from each other.

(C) Nothing is wrong with the reasoning; y increasing together with x is exactly what proportional means in a table.

(D) The table needs to have more rows added to it before anyone can decide whether it is proportional.

5. Line A passes through (0,0) and (4,8). Line B passes through (0,2) and (4,10). Both are straight lines with the same steepness. Which is proportional?

(A) Both are proportional since they're equally steep (B) Line A only

(C) Neither is proportional (D) Line B only

6. A gym charges a $10 sign-up fee plus $5 per visit. The graph of cost vs. visits is a straight line starting at (0,10). A classmate says it's proportional because doubling visits from 1 to 2 raises cost by a fixed $5. Why is your classmate wrong?

- **A** Proportional requires y/x constant AND the line through (0,0); here (0,10) breaks that, even though the $5 rate per visit is fixed.
- **B** Your classmate is right about this; a fixed rate of $5 per visit always means the relationship is proportional, no matter where the line starts.
- **C** It's non-proportional because $5 per visit is too small a rate.
- **D** The gym cost is non-proportional only because the number of visits can't actually be doubled in real life the way the classmate describes, not because of the graph.

7. A straight-line graph passes through (0,0) and (5,15). What is y/x at the point (5,15)?

A 10 **B** 1/3 **C** 20 **D** 3

8. Which straight line could be the graph of a proportional relationship?

- **A** A straight line through (0,3) and (2,9)
- **B** A straight line through (0,5) and (2,5)
- **C** A straight line through (0,0) and (2,6)
- **D** A straight line through (1,0) and (3,6)

9. A straight line passes through (1,6) and (3,10). Which statement is true?

- **A** Proportional; y gets bigger as x gets bigger
- **B** Proportional; both y-values are bigger than their x-values
- **C** Proportional; a straight line is always proportional
- **D** Not proportional; y/x is 6 at (1,6) but 10/3 at (3,10)

10. Points (2,9), (4,15), and (6,21) all lie on one straight line. Which statement is correct?

- **A** Not proportional; y/x is 4.5, then 3.75, then 3.5
- **B** Proportional; y goes up by the same 6 each time
- **C** Proportional; the points lie on a straight line
- **D** Proportional; y goes up every time x goes up

11. A recipe graph of sugar (y) vs flour (x), both in cups, is a straight line through (0,0), (4,3), and (8,6). A student finds y/x = 3/4 and says it cannot be proportional, because multiplying by a fraction less than 1 should make the answer bigger. What is the best response?

- **A** She is right; proportional unit rates must be above 1
- **B** She is wrong; y/x is 3/4 every time and the line hits (0,0)
- **C** She is wrong, but only because the graph is a straight line
- **D** She is right; a fraction means the line misses (0,0)

12. A graph shows only the part of a straight line from x = 4 to x = 10, passing through (4,10) and (8,20). The origin is not shown on the picture. Is the relationship proportional?

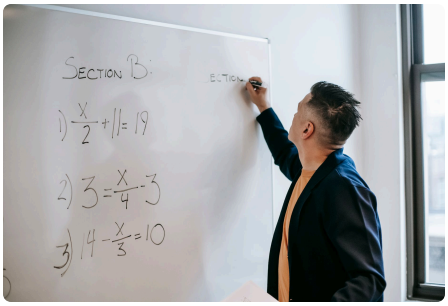
- **A** Yes; y/x is 2.5 at both points, so the line reaches (0,0)
- **B** Yes; the points make a straight line, so it is proportional
- **C** Yes; y gets bigger every time x gets bigger
- **D** No; the part shown never includes the point (0,0)

MATH · UNIT 1 · LESSON 5

Writing the equation y = kx

Goal: Write the equation y = kx for a proportional relationship given a real-world context, a table, or a graph, identifying k correctly in each case.

About 28 minutes



Teach it FOR THE PARENT

1. **Say:** Look at this: a dog walker charges $15 per hour. If y is total cost and x is hours, the equation is y = 15x. Here is what I did. I found the one number that connects x and y (15, the rate), and put it right where k goes in y = kx. That's the whole move.
   Look for: Student notices the equation is built from finding one constant rate and substituting it for k, not writing a general formula.
2. **Say:** Now you try one from a context. A printer prints 8 pages every minute. Let y = pages, x = minutes. Find k, and write the full equation y = kx with the number in place of k.
   Look for: Student states k = 8 (from the stated rate, no division needed since it's already per 1 unit) and writes y = 8x.
3. **Say:** Here's a table: x = 3, y = 12; x = 5, y = 20; x = 8, y = 32. First check that y ÷ x is the same for at least two pairs. Then write the equation.
   Look for: Student divides y/x for two pairs (12/3=4, 20/5=4), confirms constant ratio, states k = 4, writes y = 4x.
4. **Say:** Now a graph: a straight line through the origin passes through the point (6, 9). Find k using that point, and write the equation.
   Look for: Student computes k = 9 ÷ 6 = 1.5 and writes y = 1.5x, confirming the line passes through (0,0) first.
5. **Say:** Tricky one: a line passes through (0, 5) and (2, 9). Is this proportional? Can you write y = kx for it? Explain in one sentence.
   Look for: Student recognizes the line does NOT pass through the origin (y-intercept is 5, not 0), so it is not proportional and y = kx does not apply.
6. **Say:** You just wrote y = kx from a rate, a table, and a graph, and you caught one that wasn't proportional at all. Now do the practice set. Read each one carefully: is it really a constant ratio, or does it just look proportional?
   Look for: Student moves into independent practice ready to check for constant ratio and origin before writing y = kx.

If they get stuck

- *If a graph is a straight line, the relationship must be proportional.*
  Show two straight lines side by side, one through the origin and one not. Ask students to test both with the same table-ratio check (y/x constant) before deciding.
- *In a proportional table, if you just check whether y increases when x increases, that is enough to call it proportional.*
  Require students to compute y/x for every row, not just the first, and to state the ratio out loud before deciding.
- *Multiplying by a fraction less than 1 (like the unit rate 3/4) should make the answer bigger, so something is wrong.*
  Use a double number line to show the multiplication landing between two whole-number benchmarks, making the smaller result visibly reasonable.

Read together FOR THE STUDENT

A proportional relationship has one number, called k, that connects x and y every time. The rule is y = kx. Find k by dividing y by x for any pair that works: k = y ÷ x. Example: a car travels 60 miles in 1 hour, 120 miles in 2 hours. k = 60 ÷ 1 = 60, so y = 60x. Once you have k, you can write the equation without checking every single pair.

Going further: For a proportional relationship, k is the constant of proportionality: k = y/x for every point, and the equation is y = kx. From a table, pick any pair and divide y by x. From a graph, pick a clear point (not (0,0)) and divide its y by its x. From a context, k is the fixed rate, price per item, miles per hour, cups per batch. Write the equation with the actual number in place of k.

Watch it work

A recipe uses 3 cups of flour for every 2 cups of sugar. Write the equation for cups of flour (y) in terms of cups of sugar (x).

1. Identify the two matching quantities: flour = y, sugar = x.
2. Find k as the rate: k = flour ÷ sugar = 3 ÷ 2 = 1.5.
3. Write the equation with the number in place of k: y = 1.5x.
4. Check: if x = 2, y = 1.5(2) = 3. Matches the recipe.

The idea: k is just y ÷ x from one matching pair, then it goes straight into y = kx.

This table shows distance (y, miles) after x hours: (2, 70), (4, 140), (6, 210). Write the equation.

1. Check the ratio y/x stays the same for each pair: 70/2 = 35, 140/4 = 35, 210/6 = 35.
2. Because y/x is constant, this table is proportional, and k = 35.
3. Write the equation: y = 35x.
4. Check with a pair not yet used, if there is one, to confirm k still works.

The idea: Checking more than one pair confirms the ratio is truly constant, not just increasing.

A graph shows a straight line through the origin passing through the point (4, 3). Write the equation.

1. Confirm the line passes through (0,0), this is required for y = kx to apply.
2. Pick the given point (4, 3) and find k = y ÷ x = 3 ÷ 4 = 0.75.
3. Write the equation: y = 0.75x.
4. Check: multiplying x = 4 by 0.75 gives 3, which matches the point.

The idea: On a graph, k is y ÷ x from any point on the line other than the origin, and the line must pass through (0,0).

Practice: Writing the equation y = kx

1. A taxi charges $4 for every mile driven. Let y = total cost, x = miles. Write the equation.

(A) y = x + 4 (B) y = 4/x (C) y = 4x

2. A table shows: x = 2, y = 6; x = 3, y = 9; x = 4, y = 12. What is k?

(A) k = 2 (B) k = 3 (C) k = 6

3. A table shows: x = 1, y = 2; x = 2, y = 5; x = 3, y = 10. A student says this is proportional because y keeps increasing as x increases. Is the student right?

(A) Yes, since y increases every time x increases, it must be proportional

(B) No, the ratio y/x is not the same for each pair, so it is not proportional

(C) Yes, because all the numbers in the table are positive

4. A recipe uses 5 cups of water for every 4 cups of rice. Let y = cups of water, x = cups of rice. Write the equation.

(A) y = 1.25x (B) y = 0.8x (C) y = 9x

5. A student finds k = 3/4 for a unit rate, then says: 'That's wrong, multiplying by 3/4 should make my answers bigger, not smaller.' What's the mistake?

(A) Multiplying by a fraction less than 1 makes the result smaller, that's correct, not a mistake

(B) The student is right, multiplying should always make numbers bigger

(C) The student should have divided by 3/4 instead of multiplying

6. A graph shows a straight line through the origin, passing through the point (5, 2). Write the equation.

(A) y = 0.4x (B) y = 7x (C) y = 2.5x

7. A line on a graph is straight and passes through (0, 4) and (3, 10). A student says it's proportional because it's a straight line. Is this correct?

(A) Yes, because both of the coordinates in each of the two points are positive numbers, which is what proportional means

(B) No, the line does not pass through the origin (0,0), so it isn't proportional

(C) Yes, because any straight line on a graph represents a proportional relationship, wherever it starts

8. A phone plan charges based on data used, shown in a table: x = 2 GB, y = $7; x = 5 GB, y = $17.50. Write the equation for cost.

(A) y = 7x (B) y = 3.5x (C) y = 10x

9. A bakery sells muffins for $3 each. Let y = total cost and x = number of muffins. Which equation matches this situation?

(A) y = 3x (B) y = x + 3 (C) y = x/3

10. A table shows: x = 3, y = 15; x = 4, y = 20; x = 6, y = 30. What is k in y = kx?

(A) k = 1/5 (B) k = 5 (C) k = 12

11. A table shows: x = 2, y = 6; x = 4, y = 10; x = 6, y = 14. A student says this is proportional because y increases as x increases. Is the student right?

(A) Yes, because y gets bigger when x gets bigger

(B) No, because y ÷ x is 3, 2.5, and about 2.3

(C) Yes, because these points would lie on a line

12. A graph shows a straight line through the origin that also passes through the point (8, 6). Write the equation in the form y = kx.

(A) y = 4/3 x (B) y = 3/4 x (C) y = 6x

13. A deli charges by the pound for cheese. A table shows: x = 3 lb, y = $10.50; x = 8 lb, y = $28. Write the equation for cost.

(A) y = x/3.5 (B) y = x + 7.5 (C) y = 3.5x

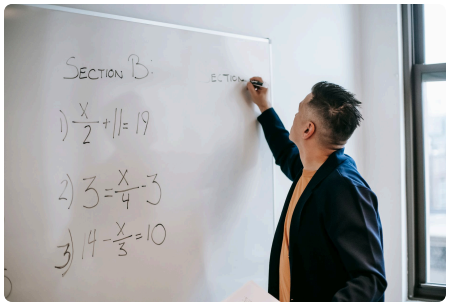
14. A drone's flight is shown in a table: x = 1/2 hour, y = 6 miles; x = 3/2 hours, y = 18 miles. Write the equation for distance y in terms of time x.

(A) y = 12x (B) y = 6x (C) y = x/12

What the point (1, r) tells us

Goal: Interpret the point (1, r) on a graph of a proportional relationship as showing the unit rate r.

About 25 minutes



Teach it FOR THE PARENT

1. **Say:** A runner's graph shows distance over time. At x = 1 minute, the graph height is 200. What does that 200 mean? Type your guess.
   Look for: Student connects the height at x=1 to speed or rate, even loosely.
2. **Say:** On a proportional graph, y = kx. Plug in x = 1: y = k. So the point (1, r) always has a y-value equal to the unit rate. This works no matter how the grid is scaled.
   Look for: Student can state that the y-value at x=1 equals the unit rate.
3. **Say:** A graph passes through (0,0) and (3, 12). What is the y-value at x = 1?
   Look for: Student finds k = 4 and says y = 4 at x = 1.
4. **Say:** Worked example: a line through (0,0) and (4,6). Step 1: find k using y = kx. Step 2: k = 1.5. Step 3: that means (1, 1.5) is on the line, the unit rate.
   Look for: Student follows each step and can restate why (1, r) works.
5. **Say:** Careful: a straight line is not always proportional. y = x + 3 is straight but does NOT pass through (0,0). Proportional lines must start at the origin.
   Look for: Student can name the origin check as the reason a line may not be proportional.
6. **Say:** New graph: grid lines spaced by 5s, line through (0,0) and (2, 15). Find the unit rate and tell me the point (1, r).
   Look for: Student computes k = 7.5 and identifies (1, 7.5), even though it's not a marked gridline.
7. **Say:** The unit rate here is 3/4. If someone says 'that seems too small for a rate,' what would you tell them?
   Look for: Student explains a unit rate can be a fraction less than 1, and that's normal.
8. **Say:** You now know: the point (1, r) always shows the unit rate. Let's practice reading it off graphs with different scales.
   Look for: Student is ready to move into independent practice items.

If they get stuck

- *If a graph is a straight line, the relationship must be proportional.*
  Show two straight lines side by side, one through the origin and one not. Ask students to test both with the same table-ratio check (y/x constant) before deciding.
- *In a proportional table, if you just check whether y increases when x increases, that is enough to call it proportional.*
  Require students to compute y/x for every row, not just the first, and to state the ratio out loud before deciding.
- *Multiplying by a fraction less than 1 (like the unit rate 3/4) should make the answer bigger, so something is wrong.*
  Use a double number line to show the multiplication landing between two whole-number benchmarks, making the smaller result visibly reasonable.

Read together FOR THE STUDENT

On a proportional graph, find the point where x = 1. Its height (the y-value) is the unit rate. Think of a car going 3/4 mile per minute. At x = 1 minute, the graph shows y = 3/4. That height IS the speed. Every point on the line has meaning, but (1, r) tells you the rate directly, no math needed, just read the height.

Going further: For a proportional relationship y = kx, the point (1, k) always sits on the graph, because y = k(1) = k. The y-coordinate at x = 1 is the unit rate, the amount y changes for one unit of x. You can read it straight off the graph, even when the grid lines are spaced by 2s or 5s instead of 1s. The value stays k. Only the picture's scale changes.

Watch it work

A graph shows a proportional line through (0,0) and (4, 6). What does the point (1, r) show, and what is r?

1. Find k using any point: y = kx, so 6 = k(4), k = 1.5.
2. The point (1, r) must have r = k, because y = k(1) = k.
3. So r = 1.5. The graph rises 1.5 units for every 1 unit right.
4. Check: at x = 1, y = 1.5. This matches the line through (0,0) and (4,6).

The idea: The y-value at x = 1 always equals the unit rate k, no matter which other point you used to find k.

A graph's grid lines are spaced by 5s. The line passes through (0,0) and (2, 15). What is the unit rate, and where does it show on the graph?

1. Find k: $y = kx$, so $15 = k(2)$, $k = 7.5$.
2. The point (1, 7.5) shows the unit rate, even though 7.5 falls between grid lines.
3. You will not see a labeled dot there, but the line still passes through that height at $x = 1$.
4. Reading the graph's scale carefully matters more than looking for a marked point.

The idea: The point (1, r) shows the unit rate even when the graph's scale hides it between grid lines.

 Name: _____

Practice: What the point (1, r) tells us

1. A proportional graph passes through (0,0) and (1, 4). What does the point (1, 4) show?

(A) The line increases forever. (B) The unit rate is 4.

(C) The relationship is not proportional.

2. A graph shows (0,0) and (1, 2/3). What is the unit rate?

(A) 2/3 (B) 1 (C) 3/2

3. A line passes through (0,0) and (5, 10). Grid lines are spaced by 2s. What is the y-value at x = 1?

(A) 5 (B) 2 (C) 10

4. A graph is straight and passes through (0, 3) and (1, 7). Is the point (1, 7) the unit rate?

(A) Yes, because the y-value went up. (B) Yes, because it's a straight line.

(C) No, because the line doesn't pass through (0,0).

5. A table shows (1,2), (2,5), (3,10). A student says this is proportional because y increases as x increases. What's wrong with that reasoning?

(A) The table needs an x = 0 row to check.

(B) Nothing is wrong. Increasing together is proof of proportionality.

(C) The ratios y/x aren't constant (2, 2.5, 3.33), so it isn't proportional.

6. A proportional graph has grid lines spaced by 10s. The line passes through (0,0) and (4, 30). What is the unit rate?

(A) 7.5 (B) 30 (C) 10

7. A unit rate is 3/4. A student multiplies 8 by 3/4 and gets 6, then says 'that seems wrong, multiplying should make it bigger.' Is the student's math correct?

(A) No, you can't multiply by a fraction.

(B) Yes, 6 is correct. Multiplying by a fraction less than 1 makes the result smaller.

(C) No, the answer should be bigger than 8, because multiplying always makes the result bigger than what you started with.

8. A graph passes through (0,0) and (6, 4). What is the y-value at x = 1, and what does it represent?

(A) 6, the x-intercept. (B) 2/3, the unit rate. (C) 4, the unit rate.

9. A proportional graph of a hiker shows hours on the x-axis and miles on the y-axis. It passes through (0,0) and (1, 3). What does the point (1, 3) tell us?

(A) The hiker walks 4 miles each hour (B) The hiker walks 1 mile every 3 hours

(C) The hiker walks 3 miles each hour

10. A proportional graph shows miles traveled (y) for hours of travel (x). You want to read the unit rate in miles per hour straight from the graph. Which point should you look at?

(A) The point where x = 1, because its height is the miles per hour

(B) The point (0, 0), because every line starts there

(C) The point where y = 1, because its x-value is the miles per hour

11. A proportional graph shows cups of flour (y) for batches of cookies (x). The line crosses x = 1 exactly halfway between the grid lines y = 2 and y = 3. What is the unit rate?

(A) 1 cup of flour per batch (B) 2 1/2 cups of flour per batch

(C) 3 cups of flour per batch

12. A toy robot's distance graph is proportional and passes through (0,0) and (1, 2/3), with seconds on the x-axis and meters on the y-axis. How many meters does it travel by x = 12 seconds?

(A) 8 meters (B) 12 2/3 meters (C) 18 meters

13. Car A's proportional graph passes through (0,0) and (1, 3/4), with minutes on the x-axis and miles on the y-axis. Car B's proportional graph passes through (0,0) and (5, 3). Which car is faster?

- **A** Car A, because 3/4 is greater than 3/5
- **B** Car B, because 3 is greater than 3/4
- **C** Same speed, since both are straight lines from (0,0)

14. A straight line graph passes through (2, 6) and (4, 10). A student says, 'It is a straight line, so it is proportional and the unit rate is 3 because 6 / 2 = 3.' What is the best response?

- **A** Proportional, since y goes up whenever x goes up
- **B** Not proportional, since 6 / 2 = 3 but 10 / 4 = 2.5
- **C** Proportional, since straight lines always show proportional relationships

Tax, tip, markup, and discount problems



Goal: Given a multi-step context involving tax, tip, markup, or discount, solve for the final amount, correctly identifying the original/base amount before calculating the percent.

About 28 minutes

Teach it FOR THE PARENT

1. **Say:** A store sells a $50 hat. It's marked up 20% for the store, then later put on sale for 20% off. Is the sale price back to $50? Take a guess before we work it out.
   Look for: Student guesses yes or no; the point is to surface the idea that markup and discount off different bases don't cancel.
2. **Say:** Every tax, tip, markup, or discount problem has a BASE, the amount the percent applies to. Convert the percent to a decimal, multiply by the base, then add (tax/tip/markup) or subtract (discount). The hard part: in multi-step problems, the base can CHANGE after step one.
   Look for: Student can state that base = what the percent is taken of, and that tax/tip/markup add while discount subtracts.
3. **Say:** Let's solve: $60 jacket, 25% off, then 8% tax on the sale price. Step 1: what's the base for the discount? Step 2: find the discount amount. Step 3: find the sale price, this becomes our new base. Step 4: find the tax on THAT new base. Step 5: add for the final price.
   Look for: Student identifies base = 60 for step 1, computes 15 discount, 45 sale price, 3.60 tax, 48.60 final, and names sale price as the new base.
4. **Say:** Quick check: in that jacket problem, was the 8% tax calculated on $60 or on $45? Why?
   Look for: Student answers $45, because that's the price after the discount, the current base.
5. **Say:** Your turn, fresh numbers: A bike costs $120. It's marked up 15% for the store. Then a customer gets 10% off the marked-up price. What's the final price? Work it step by step like we just did.
   Look for: Student finds markup amount 18, marked-up price 138, discount amount 13.80, final price 124.20, correctly using 138 as the base for the discount.
6. **Say:** You've got the moves: find the base, apply the percent, decide add or subtract, and watch for the base changing mid-problem. Time to practice on your own.
   Look for: Student is ready to move into independent items without further modeling.

If they get stuck

- *If a graph is a straight line, the relationship must be proportional.*
  Show two straight lines side by side, one through the origin and one not. Ask students to test both with the same table-ratio check (y/x constant) before deciding.
- *In a proportional table, if you just check whether y increases when x increases, that is enough to call it proportional.*
  Require students to compute y/x for every row, not just the first, and to state the ratio out loud before deciding.
- *Multiplying by a fraction less than 1 (like the unit rate 3/4) should make the answer bigger, so something is wrong.*
  Use a double number line to show the multiplication landing between two whole-number benchmarks, making the smaller result visibly reasonable.

Read together FOR THE STUDENT

Tax, tip, markup, and discount problems all work the same way. First find the base amount, the price BEFORE anything is added or taken off. Then find the percent of that base amount. Then add (for tax, tip, markup) or subtract (for discount) from the base. Example: a $40 shirt with 20% discount. Base = $40. 20% of 40 = $8. Since it's a discount, subtract: 40 - 8 = $32. The tricky part is multi-step problems, where the base changes partway through, like a discount THEN a tax on the new price.

Going further: Each of these problems has a base amount, the original price before any percent is applied. Convert the percent to a decimal, multiply by the base to get the percent amount, then add (tax, tip, markup) or subtract (discount) to get the final amount. In multi-step problems, the base changes: after a discount, the NEW price becomes the base for the next step, like calculating tax. Always ask: what number is this percent OF, right now?

Watch it work

A jacket costs $60. It is on sale for 25% off. Then 8% sales tax is added to the sale price. What is the final price?

1. Base for discount = $60 (the original price).
2. Discount amount = 0.25 × 60 = $15.
3. Sale price = 60 - 15 = $45. This is now the NEW base.
4. Tax amount = 0.08 × 45 = $3.60.
5. Final price = 45 + 3.60 = $48.60.

The idea: After a discount, the sale price becomes the base for the next percent step, not the original price.

A restaurant bill is $32. A 15% tip is added, then a $5 coupon is subtracted from that total. What is the final amount owed?

1. Base for tip = $32 (the original bill).
2. Tip amount = 0.15 × 32 = $4.80.
3. Total with tip = 32 + 4.80 = $36.80.
4. Coupon is a flat $5, not a percent, so subtract directly: 36.80 - 5 = $31.80.

The idea: Not every step in a multi-step problem is a percent, watch for flat-amount steps mixed in with percent steps.

 Name: _____

Practice: Tax, tip, markup, and discount problems

1. A shirt costs $20. Sales tax is 5%. What is the tax amount?

(A) $0.20 (B) $5.00 (C) $21.00 (D) $1.00

2. A backpack is priced at $40 and is marked down 10% for a sale. What is the sale price?

(A) $30.00 (B) $36.00 (C) $44.00 (D) $4.00

3. A meal costs $18. You leave a 20% tip. What is the total amount you pay, including tip?

(A) $14.40 (B) $3.60 (C) $20.00 (D) $21.60

4. A wholesaler buys a lamp for $25 and marks it up 40% to sell in the store. What is the store price?

(A) $15.00 (B) $10.00 (C) $65.00 (D) $35.00

5. A $80 jacket is discounted 30%. Sales tax of 5% is then added to the sale price. What is the final price?

(A) $52.00 (B) $60.00 (C) $56.00 (D) $58.80

6. A restaurant bill is $50. A 18% tip is added. Then a $10 gift card is subtracted from the total. What is the final amount owed?

(A) $40.00 (B) $47.20 (C) $49.00 (D) $59.00

7. A phone case is marked up 25% from its $16 cost to set the store price. The store then runs a 10% off sale on the store price. What is the final sale price?

(A) $18.00 (B) $20.00 (C) $16.00 (D) $14.40

8. A table is priced at $150. It's discounted 20%, then a second discount of 5% is applied to the already-discounted price for members. What is the final member price?

(A) $142.50 (B) $114.00 (C) $120.00 (D) $112.50

9. A pair of sneakers is priced at $80 and is on sale for 25% off. What is the sale price?

(A) $20 (B) $60 (C) $100 (D) $55

10. A video game costs $60. The sales tax is 8%. What is the tax amount?

(A) $55.20 (B) $4.80 (C) $0.48 (D) $64.80

11. A taxi fare is $24. You add a 15% tip. What is the total you pay, including the tip?

(A) $3.60 (B) $20.40 (C) $27.60 (D) $39.00

12. A store buys a toy for $20 and marks it up 30% to set the store price. A customer then pays 5% sales tax on the store price. What is the total the customer pays?

(A) $26.00 (B) $27.30 (C) $24.70 (D) $27.00

13. A shop buys a scarf for $40 and marks it up 50% to set the price. Later the shop takes 30% off the marked-up price. What is the final price?

(A) $42 (B) $48 (C) $18 (D) $28

14. A guitar is priced at $200. It is 10% off, and then 10% sales tax is added to the sale price. What is the final price?

(A) $180 (B) $200 (C) $162 (D) $198

MATH · UNIT 1 · LESSON 8

Does percent change order matter



Goal: Given a two-step scenario where an item is marked up then discounted (or discounted then marked up) by the same percent, determine whether the final price is the same in both orders and justify the answer using the structure of percent change, without having been shown this specific comparison during instruction.

About 28 minutes

Teach it FOR THE PARENT

1. **Say:** A shirt costs $50. The store marks it up 30%, then later discounts it 30%. Do you think it ends back at $50? Take a guess before we work it out.
   Look for: A guess, right or wrong, the goal is just to surface the intuitive answer before checking it.
2. **Say:** Percent change multiplies the CURRENT price, not the original one. Markup 30% means multiply by 1.30. Discount 30% means multiply by 0.70. Each step uses whatever price came right before it.
   Look for: Student can state that the second percent applies to the new price, not the starting price.
3. **Say:** Let's do the shirt: 50 × 1.30 = 65 after markup. Then 65 × 0.70 = 45.50 after discount. Is 45.50 the same as 50? What happened to the extra $15 we added?
   Look for: Student sees the 30% discount was taken off $65, a bigger number, so it removed more dollars than the markup added.
4. **Say:** Now flip the order for the same shirt: discount 30% first, then markup 30%. Work out both steps. Then compare your final price to 45.50.
   Look for: Student computes 50 × 0.70 = 35, then 35 × 1.30 = 45.50, and notices the two orders match here.
5. **Say:** Notice: 1.30 × 0.70 and 0.70 × 1.30 are the same multiplication, just flipped. That's why the FINAL price always matches for markup-then-discount vs. discount-then-markup, same percent.
   Look for: Student connects the matching final price to multiplication order not mattering, not to any coincidence about the shirt.
6. **Say:** You'll now see this idea with new prices and new percents I haven't shown you. Same rule applies: figure out what each step multiplies by, then compare the two full paths.
   Look for: Readiness to apply the multiply-by-factors reasoning to an unfamiliar scenario, not just recall the shirt example.

If they get stuck

- *If a graph is a straight line, the relationship must be proportional.*
  Show two straight lines side by side, one through the origin and one not. Ask students to test both with the same table-ratio check (y/x constant) before deciding.
- *In a proportional table, if you just check whether y increases when x increases, that is enough to call it proportional.*
  Require students to compute y/x for every row, not just the first, and to state the ratio out loud before deciding.
- *Multiplying by a fraction less than 1 (like the unit rate 3/4) should make the answer bigger, so something is wrong.*
  Use a double number line to show the multiplication landing between two whole-number benchmarks, making the smaller result visibly reasonable.

Read together FOR THE STUDENT

A $100 jacket goes up 20%, then down 20%. You might think you're back to $100. You're not, you end at $96. Why? The markup adds $20, giving $120. But the discount takes 20% OFF $120, which is $24, more than the $20 you added. Same percent, different order, different dollar amount, because the percent applies to a different starting price each time.

Going further: Percent change is multiplicative, not additive, so order matters. A markup then discount of the same percent p gives final = start × (1+p) × (1-p) = start × (1-p²). Reversing the order gives the same product, (1-p)(1+p), so the FINAL price is actually identical either way. What's NOT the same is the dollar amount of each step, because each percent applies to a different base price.

Watch it work

A $200 bike is marked up 10%, then discounted 10%. Find the final price.

1. Markup step: 200 × 1.10 = 220, the 10% is added to the original 200
2. Discount step: 220 × 0.90 = 198, the 10% is taken off the NEW price, 220, not the original 200
3. Compare to 200: final price 198 is less than the start, because the discount was 10% of a bigger number than the markup was

The idea: The second percent change always applies to the result of the first step, not to the original amount.

Same bike, $200, but now discount 10% first, then markup 10%. Find the final price and compare to the markup-first order.

1. Discount step: 200 × 0.90 = 180, 10% off the original 200
2. Markup step: 180 × 1.10 = 198, 10% added to the new price, 180
3. Compare orders: 198 in both cases, because 1.10 × 0.90 and 0.90 × 1.10 multiply to the same number

The idea: Swapping the order of a same-percent markup and discount gives the same final price, since multiplication order doesn't change the product, even though the in-between dollar amounts differ.

Practice: Does percent change order matter

1. A $40 hat is marked up 25%. What is the price after the markup?

(A) $45 (B) $65 (C) $50 (D) $10

2. A $60 lamp is discounted 20%. What is the sale price?

(A) $72 (B) $48 (C) $40 (D) $12

3. A $120 speaker is discounted 10%, then the new price is marked up 10%. What is the final price?

(A) $132.00 (B) $120.00 (C) $108.00 (D) $118.80

4. Which pair of multiplications gives the SAME final result, for any starting price?

(A) (1.20)(0.80) and (1.20)(1.20) (B) (1.20)+(0.80) and (0.80)+(1.20)

(C) (1.20)(0.80) and (1.00)(1.00) (D) (1.20)(0.80) and (0.80)(1.20)

5. A price is marked up 40% then discounted 40%. Which is closest to the final price, as a percent of the original?

(A) 60% of the original (B) 100% of the original (C) 116% of the original

(D) 84% of the original

6. A student says: 'Marking up 40% then discounting 40% is worse for the buyer than discounting 40% then marking up 40%.' Is this true?

(A) True, discount-first ends higher because the markup adds to a smaller number

(B) True, markup-first ends higher because the discount comes off a bigger number

(C) False, both orders give the exact same final price

7. A $50 backpack is marked up 20%. What is the price after the markup?

(A) $40 (B) $70 (C) $60 (D) $10

8. A $80 video game is discounted 25%. What is the sale price?

(A) $100 (B) $60 (C) $20 (D) $55

9. A $200 bike is marked up 10%. Then the new price is discounted 10%. What is the final price?

(A) $200 (B) $198 (C) $210 (D) $242

10. A $80 sweater is discounted 25%. Then the sale price is marked up 25%. How does the final price compare to the original $80?

(A) The same, $80 (B) $5 more than $80 (C) $15 less than $80 (D) $5 less than $80

11. A $60 jacket is marked up 25%. What percent discount on the new price brings it back to exactly $60?

(A) 15% (B) 10% (C) 20% (D) 25%

12. A $200 coat is marked up 10%, then discounted 10%, then marked up 10% again, each time from the latest price. What is the final price?

(A) $198.00 (B) $217.80 (C) $220.00 (D) $200.00

Proportions with a fixed fee



Goal: Given a completely unfamiliar real-world dataset (e.g. a utility bill with a flat fee plus a per-unit rate, presented as a table), determine whether it is proportional and, if not, identify what would need to change about the pricing structure to make it proportional.

About 28 minutes

Teach it FOR THE PARENT

1. **Say:** A rideshare app charges $3 just to open a ride, plus $2 per mile. Is that fair pricing proportional? Let's find out what 'proportional' really requires.
   Look for: Student engages with the question; no answer expected yet.
2. **Say:** Proportional means y/x is the SAME number in every row of a table. A fixed fee, like that $3 ride charge, changes the ratio every time. Watch: it makes the rule y = kx + b, and that +b is the problem.
   Look for: Student can state that y/x must stay constant, and that a fixed fee adds a +b.
3. **Say:** Quick check: table shows (2, $19), (4, $28), (6, $37) for a service. Is 19/2 the same as 28/4? Tell me both ratios.
   Look for: Student computes 9.5 and 7, notices they differ, concludes not proportional.
4. **Say:** Let's work the gym example together: $10 sign-up plus $5 per visit. Step 1: find y/x for each row. Step 2: are they equal? Step 3: find the rule y = kx + b. Step 4: what change removes the +b?
   Look for: Student follows each step and identifies removing the $10 fee as the fix.
5. **Say:** Your turn, fresh problem: a parking garage charges $4 plus $1.50 per hour. Make a table for 1, 2, 3 hours. Is it proportional? If not, what needs to change?
   Look for: Student builds table, tests ratios, finds they're unequal, and names removing the $4 fee as the fix.
6. **Say:** Now you'll see brand-new pricing setups, a ferry, a printing shop, a streaming service, that we never worked on together. Use the same test: check y/x for every row, not just whether numbers go up.
   Look for: Student is ready to apply the ratio test independently to unfamiliar contexts.

If they get stuck

- *If a graph is a straight line, the relationship must be proportional.*
  Show two straight lines side by side, one through the origin and one not. Ask students to test both with the same table-ratio check (y/x constant) before deciding.
- *In a proportional table, if you just check whether y increases when x increases, that is enough to call it proportional.*
  Require students to compute y/x for every row, not just the first, and to state the ratio out loud before deciding.
- *Multiplying by a fraction less than 1 (like the unit rate 3/4) should make the answer bigger, so something is wrong.*
  Use a double number line to show the multiplication landing between two whole-number benchmarks, making the smaller result visibly reasonable.

Read together FOR THE STUDENT

A relationship is proportional if y/x is the same number every single row. A pizza place that charges $2 per topping, no base fee, is proportional. But if they also charge a $5 box fee, that $5 breaks the pattern. Cost = 5 + 2t is NOT proportional. To fix it, the $5 fee would need to disappear. Check the ratio, not just whether numbers go up.

Going further: A table is proportional only if y/x stays constant across every row. A flat fee adds a fixed amount b, so the rule becomes y = kx + b. That b ≠ 0 breaks proportionality, even if the table's numbers rise smoothly. A straight-line graph is not proof either, it must also pass through (0,0). To make a fee-plus-rate structure proportional, the fixed fee must be removed, leaving pure y = kx.

Watch it work

A gym charges a $10 sign-up fee plus $5 per visit. Visits | Cost 1 | $15 2 | $20 3 | $25 Is this proportional? If not, what would fix it?

1. Test y/x for each row: 15/1=15, 20/2=10, 25/3≈8.33.
2. The ratios are not equal. So this table is NOT proportional.
3. Find the rule: each visit adds $5, and 1 visit costs $15, so cost = 10 + 5v.
4. The +10 is the sign-up fee. Removing that fee would make it proportional: cost = 5v.

The idea: A fixed fee added to a per-unit rate always breaks proportionality, even when cost rises steadily.

A phone plan graph is a straight line, but it does not pass through (0,0), at 0 minutes used, cost is already $8. Is this plan proportional?

1. Check the y-intercept, not just whether the graph is a straight line.
2. At x=0, y=8. For a proportional graph, x=0 must give y=0.
3. Since y=8 when x=0, this line is not proportional.
4. The rule is y = kx + 8. Removing the $8 base charge would fix it.

The idea: A straight line is only proportional if it passes through the origin, check the intercept, not just the shape.

Practice: Proportions with a fixed fee

1. A vending machine sells snacks at $1.50 each, no other fee. Table: (1,$1.50) (2,$3.00) (3,$4.50). Is this proportional?

(A) No, because the cost keeps changing (B) No, because there's no fixed fee

(C) Yes, y/x = 1.50 for every row

2. A table shows (1,$8) (2,$16) (3,$24). A student says: 'The cost goes up each time, so it's proportional.' Is that reasoning enough?

(A) No, because the numbers should decrease instead

(B) Yes, if both numbers increase together, that's the definition of proportional

(C) No, going up together doesn't prove proportional, you must check y/x is constant

3. A copy shop charges $0.75 per page, no setup fee. To find the cost of 12 pages, you multiply 12 × 0.75. Someone says 'that can't be right, multiplying should make it bigger.' What's wrong with that reasoning?

(A) They should have divided instead of multiplied

(B) The rate should have been written as the fraction 4/3 instead of 0.75, and then the multiplying would make it bigger

(C) Multiplying by a number less than 1 can make a result smaller, that's normal, not an error

4. A streaming service costs $6 setup plus $2 per movie. Table: (1,$8) (2,$10) (3,$12). Is this proportional?

(A) Yes, because the cost rises steadily by the same $2 each time as the number of movies goes up

(B) Yes, because the graph of this data would be a straight line if you plotted the three points

(C) No, the ratios (8, 5, 4) aren't equal, because of the $6 setup fee

5. A taxi charges a flat $4 plus $2 per mile. What change to this pricing would make it proportional?

(A) Remove the $2 per-mile rate, keeping only the $4 flat fee

(B) Remove the flat $4 fee, keeping only $2 per mile

(C) Keep both, since the graph is already a straight line

6. A graph shows a straight line that passes through the point (0, 5), not through the origin. A student says it must be proportional since it's a straight line. What is the flaw?

(A) There's no flaw in the student's reasoning; straight lines on a graph are always proportional no matter what

(B) A proportional graph must pass through (0,0); this one starts at (0,5), so it isn't

(C) The flaw is that lines can't actually pass through the point (0,5), so the student must have misread the graph

7. A pool membership costs $3 per visit, no other charge. Table: (2,$6) (5,$15) (10,$30). Is this proportional, and why?

(A) No, the cost values aren't all the same number

(B) Yes, y/x = 3 in every row, and there's no fixed fee

(C) Yes, the numbers increase together as visits go up

8. A parking garage charges these prices. Table (hours, cost): (1, $6) (2, $8) (3, $10). Is the cost proportional to the hours?

(A) Yes, because the cost goes up each hour

(B) No, cost divided by hours is 6, 4, 3.33, not equal

(C) Yes, because the points fall on a straight line

9. Ribbon costs $0.50 per foot, with no other fee. Mia buys 6 feet and finds 6 x 0.50 = $3. She says, 'The cost is less than 6, so I must have made a mistake.' Is Mia right?

(A) Mia is right; a rate under 1 means she should divide

(B) Mia is wrong; 0.50 of 6 feet is half of 6, which is $3

(C) Mia is right; multiplying should always make a number bigger

10. A dog walker charges a $3 booking fee plus $4 per walk. The cost is C = 3 + 4w, where w is the number of walks. Is cost proportional to the number of walks?

A. Yes, because the cost goes up with every added walk

B. No, the $3 fee means C divided by w changes

C. Yes, its graph is a straight line, so it is proportional

11. A water company's bill is shown in this table (gallons, cost): (1000, $30) (2000, $40) (3000, $50). Which change to the pricing would make the bill proportional?

A. No change needed, because the bill rises every row

B. Drop the $20 flat fee and keep $10 per 1000 gallons

C. Keep the $20 fee and raise the rate per 1000 gallons

12. An electric company posts this table (kWh used, bill): (200, $38) (400, $62) (600, $86). A customer asks whether the bill is proportional to the kWh used and, if not, what would have to change. Which answer is correct?

A. Proportional, since the points lie on a straight line

B. Proportional, since cost rises by $24 every 200 kWh

C. Not proportional; the $14 flat fee would need to go

13. A gas company offers two plans. Plan X (units, cost): (2, $7) (4, $14) (10, $35). Plan Y (units, cost): (2, $9) (4, $13) (10, $25). Which statement is correct?

A. Both are proportional, since cost rises as units rise

B. Plan X is proportional, since cost/units is 3.5 every row

C. Plan Y is proportional, since cost rises $2 per unit

Answer key

Math

Lesson 1	1. B 2. B 3. C 4. B 5. C 6. A 7. C 8. A 9. B 10. C 11. B 12. A 13. B 14. B
Lesson 2	1. A 2. A 3. A 4. C 5. A 6. A 7. C 8. C 9. A 10. A 11. A 12. B 13. B 14. A
Lesson 3	1. B 2. A 3. C 4. B 5. B 6. C 7. A 8. B 9. C 10. B 11. B 12. C 13. B 14. C
Lesson 4	1. D 2. B 3. A 4. A 5. B 6. A 7. D 8. C 9. D 10. A 11. B 12. A
Lesson 5	1. C 2. B 3. B 4. A 5. A 6. A 7. B 8. B 9. A 10. B 11. B 12. B 13. C 14. A
Lesson 6	1. B 2. A 3. B 4. C 5. C 6. A 7. B 8. B 9. C 10. A 11. B 12. A 13. A 14. B
Lesson 7	1. D 2. B 3. D 4. D 5. D 6. C 7. A 8. B 9. B 10. B 11. C 12. B 13. A 14. D
Lesson 8	1. C 2. B 3. D 4. D 5. D 6. C 7. C 8. B 9. B 10. D 11. C 12. B
Lesson 9	1. C 2. C 3. C 4. C 5. B 6. B 7. B 8. B 9. B 10. B 11. B 12. C 13. B
Unit 1 check-up	1. C 2. A 3. A 4. A 5. B 6. A 7. B 8. C 9. B 10. A 11. A 12. B 13. C 14. D 15. A 16. B 17. A 18. C

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