CLAREZA ACADEMY

Grade 6 -- Mathematics Math

Free sample: Unit 1, Lessons 1 to 8

These are 8 lessons from *Clareza Grade 6: Mathematics*, a sixth grade year a parent runs at the kitchen table. Each lesson is a short script for you, a page to read together, and a practice sheet you may copy for your household. Answers are at the back.

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Clareza Grade 6: Mathematics. Sample edition.

Clareza is a secular curriculum. Standards references: Common Core State Standards and the Next Generation Science Standards.

MATH · UNIT 1

Ratios and Rate Reasoning

This is where your child learns that comparing two quantities by 'how many times as many' gives different information than comparing by 'how many more.' They'll build ratio tables, double number lines, and move toward unit rates and percents, all as the same underlying idea shown different ways.

In this unit

1. Ratio notation and ratio language for a described relationship
2. The distinction between multiplicative (ratio) and additive (difference) comparison
3. Ratio table with equivalent ratios generated by scaling
4. Double number line as a model of a ratio relationship
5. Unit rate comparison across differently formatted representations
6. Distinctions among ratio, rate, and unit rate
7. Percent as a rate per 100, applied to an unfamiliar context
8. Combining two distinct ratio relationships into a single new relationship

MATH · UNIT 1 · LESSON 1

Ratio notation and ratio language for a described relationship

Goal: Given a real-world description (e.g. 'for every 3 cups of flour, use 2 cups of sugar'), write the relationship as a ratio using at least two notations (a:b and 'a to b').

About 25 minutes

Teach it FOR THE PARENT

1. **Say:** Here's a sentence about a classroom: 'for every 2 dogs at the park, there were 5 cats.' I write that as a ratio two ways: 2:5, and '2 to 5.' Notice dogs is named first in the sentence, so 2 (dogs) comes first in both notations. That order is the whole trick.
   Look for: Student can point to which word was named first and which number matches it.
2. **Say:** Your turn. Read this: 'for every 3 cups of flour, use 2 cups of sugar.' First, just tell me the two quantities and which one is named first, don't write the ratio yet.
   Look for: Student names flour (3) as first quantity and sugar (2) as second, matching sentence order.
3. **Say:** Now write that as colon notation: a:b. Use the order you just found.
   Look for: Student writes 3:2, not 2:3.
4. **Say:** Now write the same relationship in word form: 'a to b.'
   Look for: Student writes '3 to 2', matching the colon version exactly.
5. **Say:** Check your two answers against each other. Do the numbers appear in the same order in both? If not, fix the one that doesn't match the sentence.
   Look for: Student compares 3:2 and '3 to 2', confirms both preserve flour-first order, or corrects a mismatch.
6. **Say:** You just turned a sentence into two matching ratio notations, in the order the sentence gave you. Now you'll practice this on new descriptions, some will try to trick you into flipping the order or subtracting instead. Watch for that.
   Look for: Student is ready to move into independent practice items.

If they get stuck

- *Multiplying always makes a number bigger and dividing always makes it smaller, so scaling a ratio down by a fraction feels wrong.*
  Use a double number line where scaling down is shown as finding a point closer to zero on both lines simultaneously, making the shrinkage visible before naming it division.
- *A ratio is the same thing as a difference; comparing 'how many more' tells you the same thing as comparing 'how many times as many'.*
  Put both mixtures side by side with tape diagrams and have students test predictions by actually mixing (or simulating) the ratios to see the difference is not the same as the ratio.
- *The order of numbers in a ratio does not matter, so 3:2 and 2:3 describe the same relationship.*
  Anchor every ratio to a labeled double number line or tape diagram where the order is fixed to the label on each line; require students to name which quantity each number in the ratio refers to before writing it.

Read together FOR THE STUDENT

A ratio compares two amounts. If a recipe says 'for every 3 cups of flour, use 2 cups of sugar,' the ratio of flour to sugar is 3 to 2. You can write it two ways: 3:2 or '3 to 2.' The order matters, flour is named first, so its number comes first. Think of it like a lineup: 'flour to sugar' means flour stands first in line, sugar stands second.

Going further: A ratio is a comparison of two quantities by their order, written a:b or 'a to b.' The word order in the description tells you the number order, 'flour to sugar' means flour's number comes first. Ratios compare using multiplication, not subtraction, so '3 more' is not the same idea as '3 to 2.' Swapping the order changes the relationship, so 3:2 and 2:3 describe different mixtures.

Watch it work

A trail mix recipe says: 'for every 4 cups of nuts, use 1 cup of raisins.' Write this as a ratio in both notations.

1. Find the two quantities being compared: 4 cups of nuts, 1 cup of raisins.
2. Find the order named in the sentence: nuts is named first, raisins second.
3. Write colon notation in that order: 4:1.
4. Write word notation in that order: '4 to 1'.

The idea: The order words appear in the sentence sets the order of the numbers in the ratio.

A juice recipe says: 'mix 5 parts water with 2 parts concentrate.' Write this as a ratio in both notations.

1. Find the two quantities: 5 parts water, 2 parts concentrate.
2. Check the naming order: water first, concentrate second.
3. Write colon notation: 5:2.
4. Write word notation: '5 to 2'.
5. Note that 2:5 would describe concentrate to water instead, a different comparison.

The idea: Swapping the order describes a different relationship, so the named order must be kept exactly.

Practice: Ratio notation and ratio language for a described relationship

1. A recipe says: 'for every 3 cups of flour, use 2 cups of sugar.' Which pair correctly writes this ratio?

- (A) 3-2 and '3 minus 2'
- (B) 2:3 and '2 to 3'
- (C) 3:2 and '3 to 2'
- (D) 3:2 and '2 to 3'

2. 'For every 5 red marbles, there are 4 blue marbles.' Which is the correct ratio of red to blue?

- (A) 5:4
- (B) 4:5
- (C) 9:4

3. 'For every 6 minutes of running, Jae walks 3 minutes.' Which shows the ratio of running to walking in word form?

- (A) '3 to 6'
- (B) '3 more'
- (C) '6 to 3'

4. A paint mix says: 'for every 2 cans of blue, use 7 cans of white.' Which colon notation is correct for blue to white?

- (A) 9:2
- (B) 2:7
- (C) 5:2
- (D) 7:2

5. 'For every 8 dogs at the shelter, there are 3 cats.' A student writes the ratio as 3:8. What went wrong?

- (A) The student should have subtracted 3 from 8 to get 5:1 instead of writing the two numbers
- (B) The order was flipped, dogs was named first, so 8 should come first
- (C) Nothing, 3:8 and 8:3 mean the same thing

6. 'A garden has 12 tomato plants for every 5 pepper plants.' Which pair is correct for tomato to pepper?

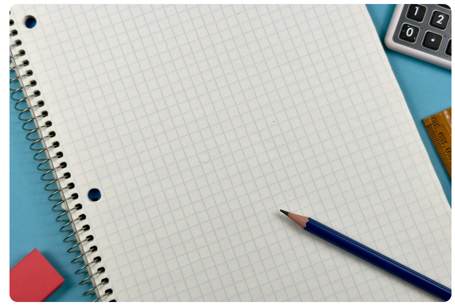
- (A) 7:5 and '7 to 5'
- (B) 12:5 and '5 to 12'
- (C) 5:12 and '5 to 12'
- (D) 12:5 and '12 to 5'

7. 'For every 9 quiet students, there are 9 loud students.' What is the ratio of quiet to loud?

- (A) 9:9
- (B) 1:1
- (C) 0:9

MATH · UNIT 1 · LESSON 2

The distinction between multiplicative (ratio) and additive (difference) comparison



Goal: Explain why a ratio and a raw count (difference) give different information about the same two quantities, using a specific example.

About 28 minutes

Teach it FOR THE PARENT

1. **Say:** Two classes both have 6 more girls than boys. Class A: 10 girls, 4 boys. Class B: 20 girls, 14 boys. Are these classes 'the same' in how they're made up? Take a guess before we dig in.
   Look for: Student notices the difference (6) is the same but is unsure if that means the classes match; may not yet mention ratio.
2. **Say:** Difference asks 'how many more.' Ratio asks 'how many times as many.' Class A: 10:4 simplifies to 5:2. Class B: 20:14 simplifies to 10:7. Same difference, different ratio, Class A has relatively more girls per boy than Class B.
   Look for: Student can restate that difference and ratio are two different questions that can give two different answers.
3. **Say:** Quick check: if two mixtures both have '3 more red beads than blue beads,' does that guarantee they have the same ratio of red to blue? Why or why not?
   Look for: Student says no, and gives or asks for numbers showing equal difference with unequal ratio.
4. **Say:** Let's walk the lemonade example together. Watch each step: I find the difference for both recipes first, then the ratio for both, then compare what each one tells us.
   Look for: Student follows along and can predict, before I say it, that the ratios might differ even though differences matched.
5. **Say:** Your turn, new numbers: Recipe C has 6 cups juice, 2 cups water. Recipe D has 12 cups juice, 8 cups water. Find the difference for each, then the ratio for each. Do they taste the same?
   Look for: Correct differences (4 and 4), correct simplified ratios (3:1 and 3:2), and the conclusion that they do not taste the same despite equal differences.
6. **Say:** In your own words: why can two pairs of numbers have the same difference but a different ratio? Use the word 'times' somewhere in your answer.
   Look for: Student connects that ratio compares by multiplying/dividing (times as many) while difference compares by subtracting, so they can move independently.
7. **Say:** Now you'll practice spotting when difference and ratio agree, disagree, and why order in a ratio matters. Read each question carefully, some are testing the same idea with new numbers.
   Look for: Student is ready to move into independent practice items.

If they get stuck

- *Multiplying always makes a number bigger and dividing always makes it smaller, so scaling a ratio down by a fraction feels wrong.*
  Use a double number line where scaling down is shown as finding a point closer to zero on both lines simultaneously, making the shrinkage visible before naming it division.
- *A ratio is the same thing as a difference; comparing 'how many more' tells you the same thing as comparing 'how many times as many'.*
  Put both mixtures side by side with tape diagrams and have students test predictions by actually mixing (or simulating) the ratios to see the difference is not the same as the ratio.
- *The order of numbers in a ratio does not matter, so 3:2 and 2:3 describe the same relationship.*
  Anchor every ratio to a labeled double number line or tape diagram where the order is fixed to the label on each line; require students to name which quantity each number in the ratio refers to before writing it.

Read together FOR THE STUDENT

A ratio compares by 'how many times.' A difference compares by 'how many more.' Example: 5 red marbles and 3 blue marbles. Difference: 2 more red than blue. Ratio: 5:3, red is not double blue. Now think of a punch mix: 10 red, 6 blue. Difference: 4 more red. Ratio: 10:6, which simplifies to 5:3, same taste as before! The difference changed, but the ratio (the taste) stayed the same. That's why recipes use ratios, not differences.

Going further: A difference (subtraction) tells you how many more of one quantity than another. A ratio (division) tells you how many times as many. These can disagree: mixture A (5 red, 3 blue) has a difference of 2 and ratio 5:3. Mixture B (10 red, 8 blue) also has difference 2, but ratio 10:8 = 5:4, a different, weaker red taste. Same difference, different ratio, different result. Two quantities can match on one measure and differ completely on the other. Always ask which comparison the situation actually needs.

Watch it work

Lemonade A uses 4 cups lemon juice and 2 cups water. Lemonade B uses 8 cups lemon juice and 6 cups water. Both have 'more lemon than water', do they taste the same? Use both the difference and the ratio to decide.

1. Find the difference for A: 4 − 2 = 2 more lemon than water.
2. Find the difference for B: 8 − 6 = 2 more lemon than water. Same difference.
3. Find the ratio for A: 4:2, which simplifies to 2:1 (divide both by 2).
4. Find the ratio for B: 8:6, which simplifies to 4:3 (divide both by 2). Different ratio.
5. A is twice as much lemon as water; B is not, B has more water relative to lemon, so B tastes weaker. The equal difference hid a real taste difference the ratio reveals.

The idea: Equal differences can hide unequal ratios, so the comparison you choose changes the conclusion you reach.

Team X won 9 more games than it lost, with a 12:3 ratio of wins to losses. Team Y also won 9 more games than it lost. Can you assume Team Y has the same win-to-loss ratio? Check using Team Y's record: 15 wins, 6 losses.

1. Team X's ratio: 12:3, which simplifies to 4:1 (divide both by 3).
2. Team X's difference: 12 − 3 = 9 more wins than losses.
3. Team Y's difference: 15 − 6 = 9 more wins than losses. Same as Team X.
4. Team Y's ratio: 15:6, which simplifies to 5:2 (divide both by 3). This is not 4:1.
5. No, matching differences (both 9) did not produce matching ratios (4:1 vs 5:2). Team X wins far more consistently per loss than Team Y.

The idea: Two situations can agree on the difference and still disagree on the ratio, because each comparison tracks something different.

Practice: The distinction between multiplicative (ratio) and additive (difference) comparison

1. Bag A has 8 red and 4 blue marbles. Bag B has 8 red and 6 blue marbles. Both bags have '4 more or fewer' comparisons in mind, which statement is TRUE?

- (A) Since both bags have 8 red marbles, their ratios of red to blue must be equal.
- (B) Both bags have the same difference between red and blue.
- (C) Bag A has a difference of 4; Bag B has a difference of 2. Their ratios also differ.

2. A recipe says '3 flour to 2 sugar.' Which ratio correctly represents flour to sugar?

(A) Either 3:2 or 2:3, since order doesn't change the comparison (B) 2:3 (C) 3:2

3. Mixture A: 5 red, 3 blue (difference of 2 more red). Mixture B: 7 red, 5 blue (difference of 2 more red). A student says these mixtures taste the same 'because both have 2 more red than blue.' Is this reasoning correct?

- (A) No, because the two mixtures have different total amounts of beads, so they cannot be compared to each other at all by any method.
- (B) No, the ratios are 5:3 and 7:5, which are not equal, so the mixtures taste different despite the equal difference.
- (C) Yes, equal differences mean the mixtures taste the same.

4. 12:8 is a ratio. A student wants to write an equivalent, smaller-numbered ratio. They say 'you can't get a smaller ratio because dividing makes numbers weird.' What should they do instead?

- (A) Subtract 4 from both numbers: 12-4=8, 8-4=4, giving 8:4.
- (B) Divide both numbers by their common factor: 12÷4=3, 8÷4=2, giving 3:2.
- (C) Leave it as 12:8, since ratios can't be simplified the way fractions can.

5. Class A: 12 girls, 6 boys. Class B: 20 girls, 14 boys. Both classes have 'more girls than boys.' Which comparison correctly shows the classes are NOT proportionally the same?

- (A) Since both classes have more girls than boys, the two ratios must be equal, so the two classes are proportionally the same.
- (B) Ratio: A simplifies to 2:1, B simplifies to 10:7, these are different, so the classes are not proportionally the same.
- (C) Difference: Class A has 6 more girls than boys, and Class B has 6 more girls than boys, so the two classes match proportionally.

6. A juice mix uses 'juice to water, 5 to 2.' Which of these ratios describes a DIFFERENT mixture (not equivalent, and not the same relationship)?

(A) 15:6 (B) 2:5 (C) 10:4

7. Two water tanks are filling. Tank X gains 3 more gallons than Tank Y gains, every hour, for several hours. Does this tell you the ratio of Tank X's total gallons to Tank Y's total gallons stays constant over time?

- (A) No, because amounts of gallons cannot be compared using ratios at all, only using differences, so the question about the ratio does not make sense for tanks.
- (B) Yes, because a constant difference of 3 gallons every hour always means a constant ratio between the two tanks, so the ratio never changes as they fill.
- (C) No, as both tanks fill up, a fixed 3-gallon gap becomes a smaller share of the growing totals, so the ratio changes (gets closer to 1:1).

Ratio table with equivalent ratios generated by scaling

Goal: Complete a ratio table by finding missing values, given one complete row and the multiplicative relationship between rows.

About 25 minutes

Teach it FOR THE PARENT

1. **Say:** Quick check first: what is 4 x 9? What is 36 / 4? Nice, your times tables are ready. Today we scale recipes up and down using those same facts.
   Look for: Correct answers 36 and 9, showing multiplication and division facts are solid before the new skill.
2. **Say:** A ratio table is rows that are all the same recipe, just bigger or smaller. Every row connects to the others by one multiply-or-divide number, called the scale factor. Find that number first. Then use it on the whole row.
   Look for: Student can restate: find the scale factor, then apply it to both numbers in a row.
3. **Say:** Look at this table: Flour 3, 6, __ and Sugar 2, 4, __. What did 3 become to get 6? Right, times 2. So what does 2 become? Now for the third column, flour is 9. What times 3 gives 9? So what happens to sugar?
   Look for: Student identifies scale factor 2, then 3, and applies each correctly: sugar 4, then sugar 6.
4. **Say:** Your turn, new numbers. Gallons: 5, 10, __. Miles: 150, __, 450. Find the scale factor from column 1 to column 2 first. Then use column 1 and column 3 together. Tell me both missing values.
   Look for: Student finds x2 for column 2 (miles = 300), and x3 for column 3 (gallons = 15).
5. **Say:** Careful with two traps. Scaling down still uses multiplication, just by a fraction-like step done through division, dividing 12:8 by 4 gives 3:2, a real smaller equivalent ratio. Also, order matters: 3 flour to 2 sugar is 3:2, never 2:3.
   Look for: Student can explain why 12:8 and 3:2 are equal, and why swapping order changes the meaning.
6. **Say:** You now know how to find a scale factor and use it to fill in a ratio table. The practice set has tables like these, find the factor first, every time, before you fill in a blank.
   Look for: Student moves into practice ready to name the scale factor before calculating.

If they get stuck

- *Multiplying always makes a number bigger and dividing always makes it smaller, so scaling a ratio down by a fraction feels wrong.*
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- *A ratio is the same thing as a difference; comparing 'how many more' tells you the same thing as comparing 'how many times as many'.*
  Put both mixtures side by side with tape diagrams and have students test predictions by actually mixing (or simulating) the ratios to see the difference is not the same as the ratio.
- *The order of numbers in a ratio does not matter, so 3:2 and 2:3 describe the same relationship.*
  Anchor every ratio to a labeled double number line or tape diagram where the order is fixed to the label on each line; require students to name which quantity each number in the ratio refers to before writing it.

Read together FOR THE STUDENT

A ratio table lists pairs of numbers that go together, like cups of juice mix to cups of water. Every row is the same recipe, just made bigger or smaller. To find a missing number, find what you multiply or divide by to get from one row to another. Then do that same thing to the other number in that row. Example: if 2:3 becomes 4:6, you multiplied by 2. So 2 became 4, and 3 becomes 6 too.

Going further: A ratio table shows equivalent ratios: pairs connected by the same scale factor. The scale factor is the number you multiply by to move from one row to another. Find it using the complete row and one known value in the new row. Then multiply the other value in that row by the same factor. Divide when moving to a smaller row. In this unit, scale factors are always whole numbers.

Watch it work

A recipe uses a ratio of 3 cups flour to 2 cups sugar. Complete the table. Flour: 3, 6, __, 12 Sugar: 2, 4, __, __

1. Find the scale factor from row 1 to row 2: 3 -> 6 means multiply by 2.
2. Apply that same factor to sugar: 2 x 2 = 4. Row 2 checks out.
3. Find flour in row 3 from row 1: 3 -> 9 is x3. So sugar is 2 x 3 = 6.
4. For row 4, flour is 12. That is 3 x 4, so sugar is 2 x 4 = 8.

The idea: One scale factor connects every row to the base row, and it applies to both numbers in that row.

A car uses 5 gallons of gas to go 150 miles. Complete the table. Gallons: 5, 10, 15, __ Miles: 150, __, __, 600

1. Check the within-row link: 150 / 5 = 30. Every mile value is 30 times the gallons.
2. For 10 gallons: 10 x 30 = 300 miles.
3. For 15 gallons: 15 x 30 = 450 miles.
4. For 600 miles: 600 / 30 = 20 gallons.

The idea: The same multiplier that links flour-to-sugar rows can also link gallons-to-miles within one row, and both routes give the same table.

Practice: Ratio table with equivalent ratios generated by scaling

1. Table: Cups juice 2, 4, __ | Cups water 5, 10, 15. What is the missing value in the top row?

(A) 10 (B) 6 (C) 7 (D) 8

2. Table: Pencils 4, __, 12 | Boxes 1, 2, 3. What is the missing number of pencils?

(A) 8 (B) 6 (C) 5 (D) 2

3. Table: Red paint 3, 6, 9 | Blue paint 5, 10, __. Find the missing blue paint value.

(A) 13 (B) 12 (C) 9 (D) 15

4. Table: Hours 2, 4, __ | Pay $18, $36, $63. What is the missing number of hours?

(A) 5 (B) 6 (C) 9 (D) 7

5. A ratio table shows 12 apples to 8 oranges. Which smaller equivalent ratio is correct?

(A) 3 apples to 2 oranges (B) You can't make it smaller, only bigger
(C) 8 apples to 4 oranges (D) 2 apples to 3 oranges

6. Table: Miles 4, __, 20 | Minutes 10, 30, __. What are the two missing values, in order (miles, minutes)?

(A) 14 miles, 40 minutes (B) 12 miles, 45 minutes (C) 12 miles, 50 minutes
(D) 10 miles, 50 minutes

7. Table: Tickets 6, 12, 18 | Dollars 15, __, 45. Find the missing dollar amount.

(A) 30 (B) 21 (C) 45 (D) 24

8. A mix uses 8 parts water to 3 parts syrup. Which table row keeps the same ratio?

(A) Water 24, Syrup 15 (B) Water 9, Syrup 24 (C) Water 11, Syrup 6
(D) Water 24, Syrup 9

Double number line as a model of a ratio relationship

Goal: Construct a double number line to represent a ratio relationship and use it to find an unknown quantity.

About 28 minutes

Teach it FOR THE PARENT

1. **Say:** Look at this double number line for 'dollars to hours worked', ratio 15:1. Top line: 0, 15, 30, 45. Bottom line: 0, 1, 2, 3, lined up under matching top marks. Notice both lines start at 0, and every matching pair stays in the same ratio. That lineup is the whole trick.
   Look for: Student can point to which feature makes it work: matching zero start, and ticks lined up so top-to-bottom always matches the ratio.
2. **Say:** Your turn. The ratio is 2 cups flour to 3 cups sugar. Draw two lines, one above the other. Label the top line and the bottom line. What do you label each one?
   Look for: Student writes two clear labels: 'flour' on top, 'sugar' on bottom, matching the order given in the ratio.
3. **Say:** Now mark the top line: 0, 2, 4, 6, 8. Mark the bottom line so each mark lines up under the matching top mark. What numbers go on the bottom line?
   Look for: Student writes 0, 3, 6, 9, 12 directly under 0, 2, 4, 6, 8, keeping equal spacing and starting both at 0.
4. **Say:** Use your double number line. Find 8 on the flour line. What sugar amount lines up under it? Write the full answer as a sentence.
   Look for: Student reads down from 8 to 12 and writes something like '8 cups flour needs 12 cups sugar.'
5. **Say:** One more line, a new ratio: 5 miles every 2 minutes. This time find miles for 8 minutes, but do it by scaling, not by counting one by one. What did you multiply by, and what is your answer?
   Look for: Student finds the scale factor (2 to 8 is times 4), multiplies 5 by 4, and states 20 miles.
6. **Say:** You just built double number lines two ways: counting up marks, and scaling by a factor. Both work every time, because matching marks always keep the same ratio. Now try the practice set on your own.
   Look for: Student moves to practice items ready to apply either method depending on the numbers given.

If they get stuck

- *Multiplying always makes a number bigger and dividing always makes it smaller, so scaling a ratio down by a fraction feels wrong.*
  Use a double number line where scaling down is shown as finding a point closer to zero on both lines simultaneously, making the shrinkage visible before naming it division.
- *A ratio is the same thing as a difference; comparing 'how many more' tells you the same thing as comparing 'how many times as many'.*
  Put both mixtures side by side with tape diagrams and have students test predictions by actually mixing (or simulating) the ratios to see the difference is not the same as the ratio.
- *The order of numbers in a ratio does not matter, so 3:2 and 2:3 describe the same relationship.*
  Anchor every ratio to a labeled double number line or tape diagram where the order is fixed to the label on each line; require students to name which quantity each number in the ratio refers to before writing it.

Read together FOR THE STUDENT

A double number line is two number lines, stacked, that both start at zero. The top line counts one thing. The bottom line counts a related thing. Example: top counts cups of juice, bottom counts cups of water, in a 2:3 mix. Line up matching marks. If top shows 2, bottom shows 3. If top shows 4, bottom shows 6. To find an unknown, line up the marks and read across.

Going further: A double number line shows a ratio as two aligned lines starting at 0. Each tick on top matches a tick on bottom. Both lines scale by the same factor. If 3:2 is the ratio, ticks go 3, 6, 9 on top and 2, 4, 6 on bottom. To find an unknown quantity, find the matching tick, or scale both lines by the same number. Order matters: top always stays the same quantity, bottom always stays the other.

Watch it work

A recipe uses a ratio of 2 cups flour to 3 cups sugar. Build a double number line. Find sugar needed for 8 cups flour.

1. Draw two lines, one above the other, both starting at 0.
2. Label the top line 'flour' and the bottom line 'sugar'.
3. Mark flour: 0, 2, 4, 6, 8. Mark sugar: 0, 3, 6, 9, 12, lined up under matching flour marks.
4. Find 8 on the flour line. Read straight down to sugar: 12.
5. So 8 cups flour needs 12 cups sugar.

The idea: Matching tick marks on both lines always stay in the same ratio, so you can read an unknown straight across.

A car travels 5 miles every 2 minutes. Use a double number line to find how far it travels in 8 minutes.

1. Draw two lines starting at 0. Label top 'minutes', bottom 'miles'.
2. Mark minutes: 0, 2, 4, 6, 8. Mark miles: 0, 5, 10, 15, 20.
3. Check the scale: minutes went from 2 to 8, which is times 4.
4. Multiply miles by 4 too: 5 x 4 = 20.
5. So in 8 minutes, the car travels 20 miles.

The idea: Scaling both lines by the same factor keeps the ratio true, even when you skip straight to a far-out value.

Practice: Double number line as a model of a ratio relationship

1. A double number line for 'apples to oranges' has top marks 0, 4, 8, 12 and bottom marks 0, 3, 6, 9. What is the ratio of apples to oranges?

(A) 4 to 3 (B) 3 to 4 (C) 1 to 1 (D) 12 to 9

2. You are building a double number line for 'ratio of red paint to blue paint, 5 red to 2 blue.' What should the top line be labeled?

(A) Blue paint (B) It doesn't matter which line (C) Red paint

3. A double number line shows 'hours' on top: 0, 3, 6, 9. It shows 'pages read' on bottom: 0, 12, 24, 36. How many pages are read in 3 hours?

(A) 36 (B) 3 (C) 12

4. Two mixtures: Mix A is 5 red to 3 blue. Mix B is 7 red to 5 blue. A student says they taste the same because both have 2 more red than blue. Is this correct reasoning for ratios?

(A) No, ratios compare by multiplying, not by 'how many more'

(B) Yes, because both mixtures use red and blue

(C) Yes, matching differences always mean matching ratios

5. A double number line for a ratio 12:8 needs a smaller equivalent pair. A student says you can't get a smaller pair because dividing makes things smaller and that 'breaks' the ratio. What is true?

(A) 12:8 has no smaller equivalent ratio

(B) You can only make ratios bigger, never smaller

(C) Dividing both numbers by the same factor gives a smaller, equal ratio: 3:2

6. A double number line shows 'liters of paint' on top and 'walls painted' on bottom. Marks: top 0, 4, 8; bottom 0, 3, 6. How many liters paint 9 walls?

(A) 12 (B) 8 (C) 9

7. A ratio is 'ratio of miles to gallons, 24 miles to 1 gallon.' A double number line has gallons on top and miles on bottom. Is this set up correctly?

- (A) Yes, it doesn't matter which quantity is on top
- (B) Yes, because gallons is a smaller number so it goes first
- (C) No, miles should be on top since it's named first

8. A recipe ratio is 9 cups water to 6 cups rice. A student wants a smaller batch and divides both by 3, getting 3:2. Another student says this ratio 'shrank' so it can't still be correct. Who is right?

- (A) Neither; ratios can't be divided at all
- (B) The first student; 3:2 is still equal to 9:6
- (C) The second student; dividing changes the ratio

MATH · UNIT 1 · LESSON 5

Unit rate comparison across differently formatted representations

Goal: Compare two rates presented in different units or formats (e.g. a table vs. a sentence) and determine which is the better buy or faster rate.

About 28 minutes

Teach it FOR THE PARENT

1. **Say:** Two vending machines. Machine A: "4 juice boxes for $5." Machine B has a little table: 6 juice boxes, $8. Which one is the better deal? Don't calculate yet, just guess.
   Look for: A guess, plus a reason like 'B has more juice' or 'A costs less total', showing they notice totals aren't enough.
2. **Say:** You can't compare totals directly because the amounts are different. You need the price for ONE juice box, that's the unit rate. Divide dollars by boxes for each machine, then compare those two numbers.
   Look for: Understanding that dividing gives an amount 'per one', which makes two different-sized deals comparable.
3. **Say:** Machine A: $5 ÷ 4 boxes. Let's do that division together, what do you get?
   Look for: Student computes $5 \div 4 = 1.25$, or gets close and I correct it.
4. **Say:** Machine B: $8 ÷ 6 boxes. Divide it, what's the unit price?
   Look for: Student computes $8 \div 6 \approx 1.33$.
5. **Say:** $1.25 per box versus $1.33 per box. Which machine is the better deal, and how do you know?
   Look for: Machine A, because a smaller price per box means cheaper overall.
6. **Say:** New problem: Trail A's sign says "hikers cover 6 miles in 2 hours." Trail B's table shows 9 miles in 3 hours. Which trail is faster, in miles per hour? Work it out step by step.
   Look for: Student divides both ($6 \div 2=3$, $9 \div 3=3$), sees they're equal, and says the trails have the same speed.
7. **Say:** Watch the order of your division. Miles per hour means miles ÷ hours, every single time. If you flip it for only one rate, your comparison breaks.
   Look for: Recognition that both rates must use the same order, quantity of interest divided by the same unit.
8. **Say:** Now you'll compare rates on your own. Some come as sentences, some as tables. Always find the unit rate first, in the same order, then compare.
   Look for: Readiness to move into independent practice items.

If they get stuck

- *Multiplying always makes a number bigger and dividing always makes it smaller, so scaling a ratio down by a fraction feels wrong.*
  Use a double number line where scaling down is shown as finding a point closer to zero on both lines simultaneously, making the shrinkage visible before naming it division.
- *A ratio is the same thing as a difference; comparing 'how many more' tells you the same thing as comparing 'how many times as many'.*
  Put both mixtures side by side with tape diagrams and have students test predictions by actually mixing (or simulating) the ratios to see the difference is not the same as the ratio.
- *The order of numbers in a ratio does not matter, so 3:2 and 2:3 describe the same relationship.*
  Anchor every ratio to a labeled double number line or tape diagram where the order is fixed to the label on each line; require students to name which quantity each number in the ratio refers to before writing it.

Read together FOR THE STUDENT

A rate compares two different things, like miles and hours. Sometimes one rate is in a table. Sometimes it's in a sentence, like "Car A drives 120 miles in 2 hours." To compare them, turn both into the same kind of number. Find how much for ONE unit, one hour, one dollar, one item. That's called a unit rate. Once both rates are unit rates, you can compare them directly. It's like comparing prices in dollars, even if one store used a chart and the other used words.

Going further: To compare two rates in different formats, first find the unit rate for each one: divide to get an amount per one unit (per hour, per dollar, per item). A table gives you pairs of numbers directly. A sentence gives you the same information in words, pull out the two numbers and divide. Once both rates are unit rates in the same units, compare the numbers. The larger unit rate wins for speed or amount; the smaller unit rate wins for price (better buy). Keep the order of your ratio consistent across both.

Watch it work

Store A: a sign says "3 bags of apples for $6." Store B's table shows 5 bags for $9. Which store has the better price per bag?

1. Store A: $6 for 3 bags. Divide 6 ÷ 3 = $2 per bag.
2. Store B: $9 for 5 bags. Divide 9 ÷ 5 = $1.80 per bag.
3. Compare unit prices: $2.00 vs $1.80.
4. Store B is cheaper per bag, so Store B is the better buy.

The idea: Different formats (sentence vs table) can hide the same kind of ratio, divide to make them match.

Runner A's times are in a table: 4 laps in 10 minutes. Runner B says, "I run 6 laps in 18 minutes." Who is faster, in minutes per lap?

1. Runner A: 10 minutes ÷ 4 laps = 2.5 minutes per lap.
2. Runner B: 18 minutes ÷ 6 laps = 3 minutes per lap.
3. Compare: 2.5 minutes per lap vs 3 minutes per lap.
4. Smaller time per lap is faster, so Runner A is faster.

The idea: For time-per-unit comparisons, the smaller unit rate wins, unlike price, check what "better" means for each context.

Practice: Unit rate comparison across differently formatted representations

1. Recipe A: a card says "2 cups flour for every 1 cup sugar." Recipe B: a table shows 6 cups flour, 3 cups sugar. Do the recipes have the same flour-to-sugar ratio?

- (A) No, because 6:3 written backward is 3:6, which is different.
- (B) No, because Recipe B uses bigger numbers.
- (C) Yes, both are 2 cups flour per 1 cup sugar.

2. A sentence says: "A car travels 100 miles in 2 hours." A table shows another car traveling 140 miles in 4 hours. Which car is faster?

- (A) The first car, at 50 miles per hour.
- (B) They are the same speed.
- (C) The second car, because 140 is a bigger number than 100.

3. Store A's sign: "5 pencils for \$2." Store B's table: 8 pencils for \$4. Which store gives a cheaper price per pencil?

- (A) Store A, at \$0.40 per pencil.
- (B) Store B, because it sells more pencils.
- (C) They cost the same per pencil.

4. Mixture A has 4 red beads and 2 blue beads. Mixture B has 9 red beads and 6 blue beads. A student says both mixtures have "2 more red than blue," so they are the same mix. Is the student right?

- (A) Yes, because both mixtures have more red beads than blue beads, so they are the same kind of mix.
- (B) Yes, because the same difference of 2 more red beads means the same mixture in both cases.
- (C) No, the ratios are different, even though the difference looks similar.

5. Painter A's job sheet: "paints 3 rooms in 6 hours." Painter B's table: 5 rooms in 8 hours. Who paints faster, in hours per room?

- (A) Painter B, at 1.6 hours per room.
- (B) Painter A, because 6 hours is less than 8 hours.
- (C) Painter A, at 1.6 hours per room.

6. A juice recipe card says "mix 3 parts water to 5 parts juice." A student needs to write this as a ratio of water to juice. Which is correct?

- (A) 5:3
- (B) It can be written either way since order doesn't matter.
- (C) 3:5

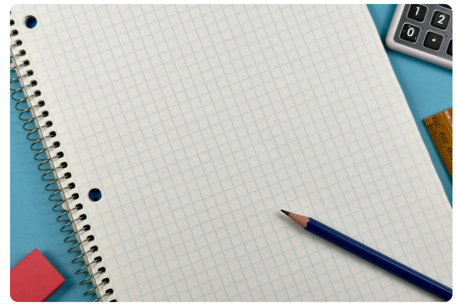
7. Phone Plan A: a table shows $30 for 6 gigabytes. Phone Plan B: a sentence says "$45 for 10 gigabytes." Which plan costs less per gigabyte?

- (A) Plan A, at $4.50 per gigabyte.
- (B) Plan A, because $30 is a smaller total price.
- (C) Plan B, at $4.50 per gigabyte.

8. A student is asked to find a smaller equivalent ratio for 12:8. They say, "You can't make it smaller by dividing, dividing only works to make ratios bigger." What should they do instead?

- (A) Divide both numbers by 4 to get 3:2, a smaller equivalent ratio.
- (B) They are right, because 12:8 cannot be made any smaller than it already is by dividing.
- (C) Add 4 to both numbers to get 16:12.

Distinctions among ratio, rate, and unit rate



Goal: Classify a given statement about a ratio relationship as expressing a ratio, a rate, or a unit rate.

About 25 minutes

Teach it FOR THE PARENT

1. **Say:** Quick question: '4 red marbles to 6 blue marbles' and '60 miles in 2 hours', are these the same KIND of statement, or different? Think for five seconds.
   Look for: Student notices one compares same-type objects (marbles) and the other compares different units (miles, hours).
2. **Say:** Every rate and unit rate IS a ratio. A ratio just compares two amounts. A rate compares two amounts with DIFFERENT units, like miles and hours. A unit rate is a rate where the second number is exactly 1, like '30 miles per 1 hour.' Order matters in all three, 3:2 is not the same as 2:3.
   Look for: Student can restate: ratio = any comparison, rate = different units, unit rate = different units AND second number is 1.
3. **Say:** Is '20 dollars for 4 tickets' a ratio, a rate, or a unit rate? Tell me which, and tell me why using the units.
   Look for: Student identifies dollars and tickets as different units (rate), and notices second number is 4, not 1, so not a unit rate.
4. **Say:** Watch me classify '5 apples to 3 oranges.' Step 1: same type of thing, no time or distance unit. Step 2: no per-one language. Step 3: that makes it a ratio. Follow along in the worked examples.
   Look for: Student can point to which step (units, then per-one check) decided the answer.
5. **Say:** Now '2 cups flour for every 1 cup sugar.' Step 1: different units of comparison, flour vs. sugar. Step 2: second number is 1. Step 3: unit rate.
   Look for: Student names the second-number-equals-1 rule as the deciding step.
6. **Say:** Your turn, fresh problem: 'A printer makes 45 pages in 1 minute.' Classify it and explain using the two checks: units, then the second number.
   Look for: Student says unit rate because minutes/pages are different units and the second number is 1.
7. **Say:** Trickier one: 'A team has 9 wins and 4 losses.' Ratio, rate, or unit rate, and why?
   Look for: Student says ratio because wins and losses are the same type of count, no differing units, no per-one statement.
8. **Say:** You now have two checks for every statement: are the units different, and is the second number 1. Use those two checks on every practice question coming up.
   Look for: Student is ready to apply the two-check method independently on new statements.

If they get stuck

- *Multiplying always makes a number bigger and dividing always makes it smaller, so scaling a ratio down by a fraction feels wrong.*
  Use a double number line where scaling down is shown as finding a point closer to zero on both lines simultaneously, making the shrinkage visible before naming it division.
- *A ratio is the same thing as a difference; comparing 'how many more' tells you the same thing as comparing 'how many times as many'.*
  Put both mixtures side by side with tape diagrams and have students test predictions by actually mixing (or simulating) the ratios to see the difference is not the same as the ratio.
- *The order of numbers in a ratio does not matter, so 3:2 and 2:3 describe the same relationship.*
  Anchor every ratio to a labeled double number line or tape diagram where the order is fixed to the label on each line; require students to name which quantity each number in the ratio refers to before writing it.

Read together FOR THE STUDENT

A ratio compares two amounts, like 4 dogs to 3 cats. A rate compares two amounts with different units, like 60 miles to 2 hours. A unit rate is a rate where the second number is 1, like 30 miles to 1 hour. Look at the words: does it just compare two groups? That's a ratio. Does it mix units like miles and hours? That's a rate. Does it say 'per one' or the second amount is 1? That's a unit rate.

Going further: A ratio compares two quantities, same or different units, in a fixed order, like 4 pens to 3 pencils. A rate is a ratio comparing two quantities with different units, like 90 words per 2 minutes. A unit rate is a rate written so the second quantity equals 1, like 45 words per minute. To classify, check the units first, then check if the second number is 1. Same units or no unit-per-unit comparison stated means ratio. Different units with a number other than 1 in the second spot means rate. Different units with 1 in the second spot means unit rate.

Watch it work

Classify: 'A fruit basket has 5 apples and 3 oranges.'

1. Check units: apples and oranges are the same TYPE of thing (fruit), and no unit like time or distance appears.
2. There is no per-one comparison stated, just two counts side by side.
3. This is a ratio: 5 apples to 3 oranges.

The idea: Same-type quantities with no per-one language make a ratio, not a rate.

Classify: 'A car travels 120 miles in 3 hours.'

1. Check units: miles and hours are different units, so this compares two different kinds of measurement.
2. Check the second number: it is 3, not 1, so it is not written per one hour yet.
3. This is a rate: 120 miles per 3 hours (a rate, not simplified to a unit rate).

The idea: Different units make it a rate; the second number staying above 1 keeps it from being a unit rate.

Classify: 'A recipe uses 2 cups of flour for every 1 cup of sugar.'

1. Check units: cups of flour and cups of sugar are different ingredients, treated as different quantities here.
2. Check the second number: it is 1 cup of sugar, so this is written per one unit.
3. This is a unit rate: 2 cups of flour per 1 cup of sugar.

The idea: A rate becomes a unit rate exactly when the second quantity is written as 1.

Practice: Distinctions among ratio, rate, and unit rate

1. A classroom has 12 boys and 15 girls. What kind of statement is this?

(A) Unit rate (B) Ratio (C) Rate

2. A car uses 8 gallons of gas to travel 240 miles. What kind of statement is this?

(A) Ratio (B) Rate (C) Unit rate

3. A runner covers 6 miles in 1 hour. What kind of statement is this?

(A) Ratio (B) Rate (C) Unit rate

4. A recipe calls for 3 cups of flour to 2 cups of sugar. A student says this is the same as 2 cups of flour to 3 cups of sugar because the numbers are the same. Which statement is true?

(A) Both are rates, so order doesn't matter.

(B) The student is wrong; 3 flour to 2 sugar is a different ratio from 2 flour to 3 sugar.

(C) The student is right; order doesn't matter in a ratio.

5. A store sells 5 shirts for 25 dollars. What kind of statement is this?

(A) Rate (B) Unit rate (C) Ratio

6. Mixture A has 5 red beads and 3 blue beads. Mixture B has 7 red beads and 5 blue beads. A student says both mixtures have the 'same ratio' because both have 2 more red beads than blue beads. What is the error?

(A) There is no error; both mixtures do have the same ratio.

(B) The student compared differences (how many more) instead of comparing the ratio itself.

(C) The student should have called these two mixtures rates instead of ratios, since beads are measured.

7. A student simplifies the ratio 12:8 down to 3:2 by dividing both numbers by 4. Another student says this can't be right because dividing makes numbers smaller, and a ratio 'shrinking' doesn't make sense. What is wrong with the second student's thinking?

- (A) Dividing both parts of a ratio by the same number keeps the relationship equivalent, even though the numbers get smaller.
- (B) The second student is right; you can only add or subtract to simplify a ratio.
- (C) 12:8 and 3:2 are actually different ratios, so the second student is correct.

8. A painter mixes 9 liters of blue paint with 3 liters of white paint. What kind of statement is this?

(A) Ratio (B) Rate (C) Unit rate

MATH · UNIT 1 · LESSON 7

Percent as a rate per 100, applied to an unfamiliar context

Goal: Convert a ratio to a percent and interpret percent as a rate per 100 in a context never used during instruction (e.g. a sports statistic or a survey result from a different domain).

About 28 minutes

Teach it FOR THE PARENT

1. **Say:** Quick retrieval: what is 30% written as a ratio out of 100? And what does 'percent' actually mean, in one phrase?
   Look for: Student says 30:100 and defines percent as 'per hundred' or 'out of 100'.
2. **Say:** Every ratio a:b can become a percent. Find the scale factor that turns b into 100. Then multiply a by that same factor. That's it, one move, every time.
   Look for: Student can restate: find the factor that makes the second number 100, then apply it to the first number too.
3. **Say:** Watch this: 26 out of 40 people liked a snack in a survey. What's the ratio? What factor turns 40 into 100? Now apply that factor to 26, what percent do we get?
   Look for: Student identifies 26:40, computes scale factor 2.5, applies it to get 65:100, states 65%.
4. **Say:** Your turn, no help this time: a store had 250 customers, 55 used a coupon. Write the ratio, find the scale factor, and tell me the percent.
   Look for: Student writes 55:250, finds factor 0.4, computes 22, states 22%.
5. **Say:** Here's a curveball: a soccer player took 8 shots and scored 5. Is 5:8 the same relationship as 8:5? Why does order matter here?
   Look for: Student says no, 5:8 is goals-per-shots, 8:5 would flip the meaning; order encodes which number is which quantity.
6. **Say:** You now have one method for turning any ratio into a percent, in any situation, sports, surveys, money, anything. The practice set will test that on situations you have not seen before. Use the scale-factor method every time.
   Look for: Student is ready to apply the method to unfamiliar contexts without a cue for which method to use.

If they get stuck

- *Multiplying always makes a number bigger and dividing always makes it smaller, so scaling a ratio down by a fraction feels wrong.*
  Use a double number line where scaling down is shown as finding a point closer to zero on both lines simultaneously, making the shrinkage visible before naming it division.
- *A ratio is the same thing as a difference; comparing 'how many more' tells you the same thing as comparing 'how many times as many'.*
  Put both mixtures side by side with tape diagrams and have students test predictions by actually mixing (or simulating) the ratios to see the difference is not the same as the ratio.
- *The order of numbers in a ratio does not matter, so 3:2 and 2:3 describe the same relationship.*
  Anchor every ratio to a labeled double number line or tape diagram where the order is fixed to the label on each line; require students to name which quantity each number in the ratio refers to before writing it.

Read together FOR THE STUDENT

A percent is a ratio compared to 100. If a basketball player makes 18 out of 25 shots, ask: how many would that be out of 100? Scale the ratio 18:25 up to a ratio out of 100. Multiply both numbers by 4, since 25 times 4 is 100. That gives 72:100, so the player makes 72% of shots. The trick works for ANY ratio, even ones you have never seen before, money, shots, surveys, anything.

Going further: Percent means 'per 100', it is a ratio with 100 as the second term. To convert any ratio a:b to a percent, find the scale factor that turns b into 100, then apply that same factor to a. Example: 9:20, scale factor is 5 (20 × 5 = 100), so 9 × 5 = 45, giving 45%. This works whether b is a whole-number factor of 100 or not; you can also divide a by b and multiply by 100. The context (money, sports, surveys) never changes the method.

Watch it work

A survey asked 40 people if they liked a new snack. 26 said yes. What percent said yes?

1. Write the ratio of yes-answers to total people: 26:40.
2. Find the scale factor that turns 40 into 100. 100 ÷ 40 = 2.5.
3. Multiply both parts of the ratio by 2.5: 26 × 2.5 = 65, and 40 × 2.5 = 100.
4. The equivalent ratio is 65:100, so 65% of people said yes.

The idea: Any ratio a:b becomes a percent by scaling b to 100 and applying the same factor to a.

A store had 250 customers, and 55 of them used a coupon. What percent used a coupon?

1. Write the ratio of coupon-users to total customers: 55:250.
2. Find the scale factor that turns 250 into 100. $100 \div 250 = 0.4$.
3. Multiply both parts by 0.4: $55 \times 0.4 = 22$, and $250 \times 0.4 = 100$.
4. The equivalent ratio is 22:100, so 22% of customers used a coupon.

The idea: The scale factor can be less than 1 when the starting number is bigger than 100, it still works the same way.

Practice: Percent as a rate per 100, applied to an unfamiliar context

1. A recipe ratio is 3 cups flour to 2 cups sugar. What is the ratio of sugar to flour?

(A) 5:2 (B) 3:2 (C) 2:3

2. Simplify the ratio 12:8 to lowest terms by dividing both numbers.

(A) 4:0 (B) You can't make 12:8 smaller, only bigger (C) 3:2

3. Convert the ratio 9:20 to a percent. What scale factor turns 20 into 100?

(A) 80 (B) 5 (C) 11

4. Using the scale factor from 9:20, what percent is equivalent to this ratio?

(A) 45% (B) 9% (C) 20%

5. Two mixtures: A has 5 red, 3 blue paint parts. B has 7 red, 5 blue paint parts. Do they make the same shade?

(A) Yes, because both have 2 more red than blue

(B) Yes, because both mixtures use red and blue

(C) No, because 5:3 and 7:5 are not equivalent ratios

6. A class has 250 students, and 55 ride the bus. What percent ride the bus?

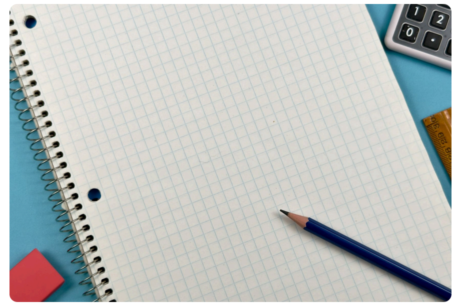
(A) 55% (B) 195% (C) 22%

7. A survey of 16 people found 12 prefer tea over coffee. What percent prefer tea?

(A) 75% (B) 4% (C) 12%

MATH · UNIT 1 · LESSON 8

Combining two distinct ratio relationships into a single new relationship



Goal: Given a non-routine problem where two ratios must be reasoned about simultaneously (e.g. mixing two batches with different ratios into one), plan a solution strategy and justify why it works.

About 30 minutes

Teach it FOR THE PARENT

1. **Say:** You have two jugs of lemonade. Jug A is strong: 1 scoop mix per 2 cups water. Jug B is weak: 1 scoop mix per 5 cups water. You pour both jugs into one big pitcher. Is the pitcher strong, weak, or something new? Take a guess before we work it out.
   Look for: Any answer is fine here. Look for a guess that treats the combined mix as different from both originals, not just picking A or B.
2. **Say:** Here is the key rule: when you combine two ratios, you cannot average the ratio numbers. You have to find REAL amounts in each batch, add the matching amounts together, then build a new ratio from those totals.
   Look for: Student can restate: find real amounts first, add matching parts, then form the new ratio.
3. **Say:** Quick check: Mix A is 1 red to 4 white, and you use 5 cups of Mix A. How many cups are red, and how many are white?
   Look for: 1 cup red, 4 cups white, from recognizing 5 cups splits into 1 part per 5.
4. **Say:** Let's solve the paint problem together, step by step. Paint A is 1:3 blue to white, 4 cups total. Paint B is 1:5 blue to white, 6 cups total. Watch how I find real amounts in each, then add blue to blue and white to white.
   Look for: Student follows each step: split A into 1 blue/3 white, split B into 1 blue/5 white, add to get 2:8, simplify to 1:4.
5. **Say:** Before we move on: why can't we just average 1:3 and 1:5 to get the answer?
   Look for: Student notices the batches are different sizes, so simple averaging ignores how much of each was actually mixed in.
6. **Say:** Your turn, a new problem: Trail mix A is 2 nuts to 3 raisins. Trail mix B is 1 nut to 2 raisins. You combine 6 cups of A with 3 cups of B. Find the nut-to-raisin ratio of the combined mix. Show your steps.
   Look for: Splits A into nuts/raisins using its ratio and 6 cups, splits B using its ratio and 3 cups, adds matching parts, forms final ratio.
7. **Say:** You now have a plan for any mixture problem: get real amounts, add matching quantities, rebuild the ratio. The practice set ahead uses this same plan on new situations. Some will not look like paint or food at all.
   Look for: Readiness to apply the three-step plan without being told which numbers to combine.

If they get stuck

- *Multiplying always makes a number bigger and dividing always makes it smaller, so scaling a ratio down by a fraction feels wrong.*
  Use a double number line where scaling down is shown as finding a point closer to zero on both lines simultaneously, making the shrinkage visible before naming it division.
- *A ratio is the same thing as a difference; comparing 'how many more' tells you the same thing as comparing 'how many times as many'.*
  Put both mixtures side by side with tape diagrams and have students test predictions by actually mixing (or simulating) the ratios to see the difference is not the same as the ratio.
- *The order of numbers in a ratio does not matter, so 3:2 and 2:3 describe the same relationship.*
  Anchor every ratio to a labeled double number line or tape diagram where the order is fixed to the label on each line; require students to name which quantity each number in the ratio refers to before writing it.

Read together FOR THE STUDENT

You know how to make one ratio bigger or smaller. This skill asks: what happens when you pour TWO different ratio mixtures into ONE container? Think of two paint mixes. Mix A is 1 blue to 2 white. Mix B is 1 blue to 4 white. If you dump both into one bucket, the new ratio is NOT just 1:2 or 1:4. You have to count the total blue and total white separately, then compare those totals. The trick is to find real amounts for each mixture first, then add matching parts together.

Going further: A mixture problem combines two ratio relationships into a new one. Each original ratio stays fixed inside its own batch. To combine them, convert each ratio to real amounts, using a common scale if needed. Then add the like quantities across both batches: total of ingredient X, total of ingredient Y. The new ratio compares those two totals, not the original ratio numbers. This is why you cannot average or add the ratios directly. You must add real quantities first, then re-form the ratio from those sums.

Watch it work

Paint A is mixed 1 part blue to 3 parts white. Paint B is mixed 1 part blue to 5 parts white. You mix 4 cups of Paint A with 6 cups of Paint B. What is the ratio of blue to white in the new mix?

1. Find blue and white in Paint A: 4 cups total, ratio 1:3 means 4 parts total, so 1 cup blue, 3 cups white.
2. Find blue and white in Paint B: 6 cups total, ratio 1:5 means 6 parts total, so 1 cup blue, 5 cups white.
3. Add matching quantities: total blue = 1 + 1 = 2. Total white = 3 + 5 = 8.
4. Write the new ratio from the totals: blue to white is 2:8, which simplifies to 1:4.

The idea: Convert each ratio to real amounts first, then add matching quantities before forming the new ratio.

Recipe A uses 2 cups flour for every 1 cup sugar. Recipe B uses 3 cups flour for every 1 cup sugar. You combine 1 batch of A with 2 batches of B. What is the flour-to-sugar ratio of the combined dough?

1. Scale Recipe A by 1 batch: 2 cups flour, 1 cup sugar.
2. Scale Recipe B by 2 batches: flour = 3 x 2 = 6 cups, sugar = 1 x 2 = 2 cups.
3. Add matching quantities: total flour = 2 + 6 = 8. Total sugar = 1 + 2 = 3.
4. The combined ratio is flour to sugar = 8:3. This cannot be simplified further.

The idea: Scaling each recipe by its batch count first is what makes the totals add up correctly.

Practice: Combining two distinct ratio relationships into a single new relationship

1. Fruit punch A is 1 part juice to 4 parts soda. You use 10 cups of Punch A. How many cups are juice?

- (A) 4 cups
- (B) 2 cups
- (C) 1 cup
- (D) 6 cups

2. Ratio 12:8 is simplified by dividing both numbers by 4. What is the simplified ratio?

- (A) 2:3
- (B) 3:2
- (C) 8:4
- (D) 16:12

3. Mixture A has 5 red to 3 blue tiles. Mixture B has 7 red to 5 blue tiles. A student says both mixtures have the same red-to-blue relationship because both have 2 more red than blue. Is this reasoning correct?

- (A) No, because ratio compares by multiplication, not by the difference between numbers
- (B) Yes, because both differences equal 2, so the ratios of red to blue tiles are equal in Mixture A and Mixture B
- (C) Yes, because the order of the numbers in a ratio does not matter here, so 5:3 and 7:5 are the same relationship
- (D) No, but only because red should always be listed first in the ratio, and the student did not say which color came first

4. Cleaner A is 1 part bleach to 6 parts water. You use 7 cups of Cleaner A. How many cups are bleach and how many are water?

- (A) 3.5 cups bleach, 3.5 cups water
- (B) 1 cup bleach, 6 cups water
- (C) 2 cups bleach, 5 cups water
- (D) 6 cups bleach, 1 cup water

5. Juice Mix A is 2 parts concentrate to 3 parts water. You make 3 batches of Mix A. How many total parts of concentrate and water result?

- (A) 6 parts concentrate, 9 parts water
- (B) 5 parts concentrate, 6 parts water
- (C) 2 parts concentrate, 3 parts water
- (D) 9 parts concentrate, 6 parts water

6. Tea A is 1 part syrup to 4 parts water, made with 5 cups total. Tea B is 1 part syrup to 9 parts water, made with 10 cups total. You combine both teas. What is the combined syrup-to-water ratio?

- (A) 1:6.5
- (B) 2:9
- (C) 2:13
- (D) 13:2

7. You plan to combine two salt-water mixes. Mix A is 1 part salt to 5 parts water. Mix B is 1 part salt to 2 parts water. Which first step correctly starts a valid plan?

- (A) Assume that both mixes taste exactly the same since both of them start with 1 part salt, so no plan is needed for combining them
- (B) Add 1+1 for the salt and 5+2 for the water directly from the ratio numbers, which gives 2 parts salt to 7 parts water for the combined mix
- (C) Find how many cups of salt and water are in a chosen amount of each mix, using its own ratio
- (D) Average 1:5 and 1:2 to get one combined ratio

Answer key

Math

Lesson 1	1. C 2. A 3. C 4. B 5. B 6. D 7. A
Lesson 2	1. C 2. C 3. B 4. B 5. B 6. B 7. C
Lesson 3	1. B 2. A 3. D 4. D 5. A 6. C 7. A 8. D
Lesson 4	1. A 2. C 3. C 4. A 5. C 6. A 7. C 8. B
Lesson 5	1. C 2. A 3. A 4. C 5. A 6. C 7. C 8. A
Lesson 6	1. B 2. B 3. C 4. B 5. A 6. B 7. A 8. A
Lesson 7	1. C 2. C 3. B 4. A 5. C 6. C 7. A
Lesson 8	1. B 2. B 3. A 4. B 5. A 6. C 7. C
Unit 1 check-up	1. B 2. A 3. B 4. B 5. A 6. C 7. B 8. A 9. B 10. B 11. C 12. A 13. B 14. C 15. B 16. A

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