CLAREZA ACADEMY

Grade 3 -- Mathematics Math

Free sample: Unit 1, Lessons 1 to 8

These are 8 lessons from *Clareza Grade 3: Mathematics*, a third grade year a parent runs at the kitchen table. Each lesson is a short script for you, a page to read together, and a practice sheet you may copy for your household. Answers are at the back.

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Clareza Grade 3: Mathematics. Sample edition.

Clareza is a secular curriculum. Standards references: Common Core State Standards and the Next Generation Science Standards.

MATH · UNIT 1

Equal Groups, Arrays, and the Meaning of Multiplication

This unit gives a name and a picture to something your child already half-knows: counting things in equal groups. They'll learn two pictures - equal groups and arrays (rows and columns of dots or tiles) - and the words that go with them: factor, product, row, column. The array picture is the star here because rotating it shows, without any rule to memorize, that 4 groups of 6 and 6 groups of 4 come out the same.

In this unit

1. Equal-groups pictures as a model of multiplication
2. Arrays: rows, columns, and total
3. Factor and product vocabulary
4. Commutative property shown by rotating an array
5. Equal-groups and array pictures as two representations of one fact
6. Equal-groups word problems within 100
7. Limits of the repeated-addition analogy for multiplication
8. Commutative property as evidenced by array rotation

MATH · UNIT 1 · LESSON 1

Equal-groups pictures as a model of multiplication

Goal: Given a picture of equal groups (e.g., 4 baskets of 3 apples), the student writes the total as repeated addition and as a multiplication equation.

About 18 minutes

Teach it FOR THE PARENT

1. **Say:** I have 5 boxes. Each box has 4 crayons. How many crayons in all? I could count one by one, or find a faster way. Let's find that faster way today.
   Look for: Student attends to the idea of a faster way, not a specific number answer yet.
2. **Say:** Look at this picture: 2 plates, 6 cookies on each. I write the addition first: 6 + 6 = 12, one 6 for each plate. Then I write the equation: 2 x 6 = 12. The 2 is the number of plates. The 6 is cookies on one plate.
   Look for: Student can point to which number in the equation is the group count and which is the group size.
3. **Say:** Now look at your own picture: 3 baskets, 4 apples in each. Write the repeated addition. Add the group size once for every group. How many times will you write the 4?
   Look for: Student writes 4 + 4 + 4, with exactly three 4s, one per basket.
4. **Say:** Use that same picture. Now write the multiplication equation. Put the number of groups first, then the group size, then the total.
   Look for: Student writes 3 x 4 = 12, groups x group size = total, matching the picture.
5. **Say:** Here's a new picture: an array with 5 rows, 2 dots in each row. Rows are groups too. Write the addition, then the equation.
   Look for: Student treats rows as equal groups, writes 2+2+2+2+2=10, then 5 x 2 = 10.
6. **Say:** You just turned three pictures into addition and equations. Now you'll do it on your own with new baskets, boxes, and arrays. Count groups first, then group size, every time.
   Look for: Student is ready to apply the same two-step routine without the model in front of them.

If they get stuck

- *Multiplying always makes a number bigger.*
  On Day 14 explicitly flag: this is true for whole numbers greater than 1, but not always true once fractions arrive - do not resolve it fully, just name the boundary.
- *Rotating an array changes how many dots there are, because it looks different.*
  Have students physically rotate a card of counters and count before and after, side by side, several times before naming the property.
- *3 groups of 4 and 4 groups of 3 are different facts that must be memorized separately.*
  Show the same array as both 'read by rows' and 'read by columns' descriptions before introducing the word commutative.

Read together FOR THE STUDENT

A picture can show equal groups, like 4 baskets with 3 apples in each. To find the total, you can add: 3+3+3+3. You can also multiply: 4 x 3. Multiplication is a fast way to add the same number over and over. The number of groups and the size of each group both matter.

Going further: Equal groups are groups that all have the same number of things. When you see equal groups, you can write repeated addition, like 3+3+3+3, adding the group size once per group. You can also write a multiplication equation: number of groups x group size = total. Both ways give the same total, because multiplication is repeated addition written faster.

Watch it work

A picture shows 3 baskets. Each basket has 5 apples. Write the total as repeated addition and as a multiplication equation.

1. Count the groups: there are 3 baskets, so there are 3 equal groups.
2. Count how many are in one group: each basket has 5 apples.
3. Write repeated addition: add 5 three times, once per basket -> 5 + 5 + 5 = 15.
4. Write the multiplication equation using groups x group size: 3 x 5 = 15.

The idea: The number of groups tells you how many times to add the group size.

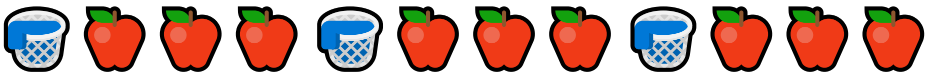
A picture shows 4 rows of 2 stars, arranged as an array. Write the total as repeated addition and as a multiplication equation.

1. Count the rows: there are 4 rows, so there are 4 equal groups.
2. Count how many stars are in one row: each row has 2 stars.
3. Write repeated addition: 2 + 2 + 2 + 2 = 8.
4. Write the multiplication equation: 4 x 2 = 8.

The idea: An array's rows are equal groups too, so the same group-counting method works.

Practice: Equal-groups pictures as a model of multiplication

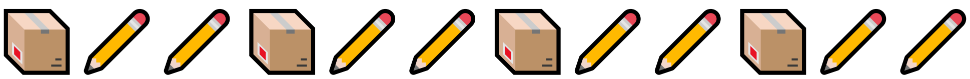
1. How many baskets are there, and how many apples in each?



3 baskets, 3 apples in each

(A) 3 baskets, 9 apples each (B) 9 baskets, 3 apples each (C) 3 baskets, 3 apples each

2. A picture shows 4 boxes with 2 pencils in each box. Which repeated addition matches it?



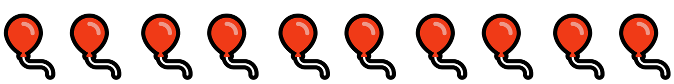
4 boxes, 2 pencils in each

(A) 2 + 2 + 2 + 2 (B) 4 + 4 + 2 (C) 2 + 4

3. A picture shows 3 rows of 5 dots in an array. Which multiplication equation matches it?

(A) 5 x 3 x 5 (B) 3 x 5 = 15 (C) 3 + 5 = 8

4. A picture shows 5 groups of 2 balloons. Which repeated addition and equation both match?



5 groups of 2 balloons

(A) 5+5 = 10 and 2 x 5 = 10 (B) 2+2+2+2 = 8 and 4 x 2 = 8

(C) 2+2+2+2+2 = 10 and 5 x 2 = 10

5. An array has 4 rows of 3 dots. Someone turns the paper sideways. Now how many dots are there?

(A) Still 12 dots (B) Now 3 dots, because it looks like fewer rows

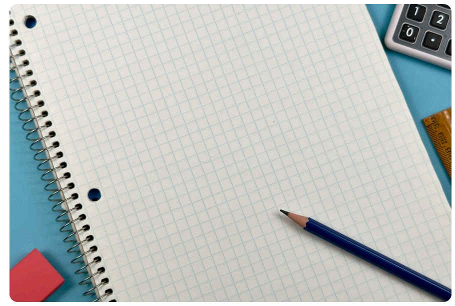
(C) You cannot tell without recounting every dot

6. Mia says 3 groups of 4 and 4 groups of 3 are two different facts to memorize. Is she right?

- (A) Yes, because the pictures look different
- (B) Yes, because 3 x 4 and 4 x 3 give different totals
- (C) No, both give the same total, 12

MATH · UNIT 1 · LESSON 2

Arrays: rows, columns, and total



Goal: The student builds an array with a stated number of rows and columns and states the total.

About 18 minutes

Teach it FOR THE PARENT

1. **Say:** Think of a muffin pan. It has rows going across. It has columns going up and down. Today we build arrays like that and count the total.
   Look for: Student can point to a row and a column in a picture.
2. **Say:** A row goes side to side. A column goes up and down. To build an array, you place the same number in every row. Then you count everything for the total.
   Look for: Student repeats that rows and columns are different directions.
3. **Say:** Watch me build 3 rows of 4. I draw 3 rows. Each row gets 4 dots. Now I count all 12 dots one by one. The total is 12. Did I put 4 in every row?
   Look for: Student confirms each row has 4 dots before agreeing the total is 12.
4. **Say:** Now watch a trick. I turn my array on its side. It looks different now. But did any dots disappear? Count again and check.
   Look for: Student recounts and sees the total is still 12, addressing the rotation misconception.
5. **Say:** Your turn. Build an array with 4 rows and 3 columns. Draw it, then count the total. Tell me each step out loud.
   Look for: Student draws 4 equal rows of 3, counts all objects, and states the total is 12.
6. **Say:** You can now build any array from rows and columns and find the total by counting. Let's practice this with new arrays.
   Look for: Student is ready to move to independent practice items.

If they get stuck

- *Multiplying always makes a number bigger.*
  On Day 14 explicitly flag: this is true for whole numbers greater than 1, but not always true once fractions arrive - do not resolve it fully, just name the boundary.
- *Rotating an array changes how many dots there are, because it looks different.*
  Have students physically rotate a card of counters and count before and after, side by side, several times before naming the property.
- *3 groups of 4 and 4 groups of 3 are different facts that must be memorized separately.*
  Show the same array as both 'read by rows' and 'read by columns' descriptions before introducing the word commutative.

Read together FOR THE STUDENT

An array is rows and columns of objects, like a muffin pan. Rows go side to side. Columns go up and down. If you build 3 rows with 4 in each row, you made an array. Count every object to find the total. It does not matter if you count by rows or by columns. The total stays the same either way.

Going further: An array is objects lined up in rows and columns. A row goes across. A column goes up and down. To build an array with stated rows and columns, place that exact number in each row. Then count all objects for the total. Turning the array does not change the total. It only swaps which dimension is rows and which is columns.

Watch it work

Build an array with 3 rows and 4 columns. Find the total.

1. Draw 3 rows. Each row will hold the same number of dots.
2. Put 4 dots in each row, lined up in columns.
3. Count all the dots, one by one.
4. The total is 12 dots.

The idea: Rows tell you how many groups. Columns tell you how many are in each group.

Build an array with 5 rows and 2 columns. Find the total.

1. Draw 5 rows, stacked one under the next.
2. Put 2 dots in each row.
3. Count every dot across all 5 rows.
4. The total is 10 dots.

The idea: Counting every dot always gives the true total, no matter the shape.

Practice: Arrays: rows, columns, and total

1. Build an array with 2 rows and 3 columns. What is the total?

(A) 6 (B) 8 (C) 5

2. Build an array with 3 rows and 3 columns. What is the total?

(A) 9 (B) 6 (C) 12

3. Build an array with 4 rows and 2 columns. What is the total?

(A) 8 (B) 6 (C) 10

4. An array has 3 rows of 5 dots. Someone turns it sideways. It now looks like 5 rows of 3. What is the new total?

(A) 8 (B) 20 (C) 15

5. Build an array with 6 rows and 2 columns. What is the total?

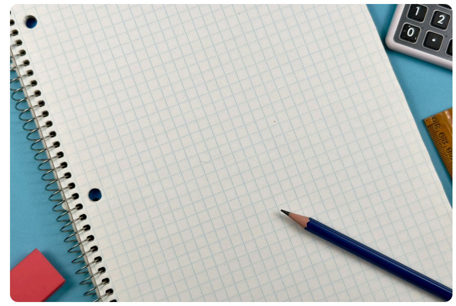
(A) 12 (B) 8 (C) 14

6. Mia says 3 rows of 4 and 4 rows of 3 must have different totals. Is she right?

(A) No, both totals are 12 (B) Yes, they are different totals

(C) We cannot tell without more information

Factor and product vocabulary



Goal: The student names the factors and product in a given multiplication equation and in a matching array.

About 18 minutes

Teach it FOR THE PARENT

1. **Say:** Look at 6 x 3 = 18. Two of these numbers have a special job. They get multiplied. Can you point to them?
   Look for: Student points to 6 and 3, not 18.
2. **Say:** The numbers you multiply are called factors. The answer is called the product. In 6 x 3 = 18, 6 and 3 are factors. 18 is the product.
   Look for: Student repeats which numbers are factors and which is the product.
3. **Say:** Look at this array: 4 rows, 5 columns, 20 dots total. Rows and columns are factors. Total dots are the product. So 4 and 5 are factors. 20 is the product.
   Look for: Student can point to rows as one factor and columns as the other.
4. **Say:** In 7 x 2 = 14, which numbers are the factors?
   Look for: Student says 7 and 2, not 14.
5. **Say:** Here is a new array: 3 rows, 6 columns, 18 dots. Tell me the two factors and the product.
   Look for: Student says factors are 3 and 6, product is 18.
6. **Say:** If you turn that array on its side, it has 6 rows and 3 columns now. The dot count is still 18. Turning it does not change how many dots there are.
   Look for: Student agrees the product stays 18 after rotation.
7. **Say:** Now you will practice naming factors and products in equations and arrays. Some arrays will look turned. Count carefully before you answer.
   Look for: Student is ready to start independent practice.

If they get stuck

- *Multiplying always makes a number bigger.*
  On Day 14 explicitly flag: this is true for whole numbers greater than 1, but not always true once fractions arrive - do not resolve it fully, just name the boundary.
- *Rotating an array changes how many dots there are, because it looks different.*
  Have students physically rotate a card of counters and count before and after, side by side, several times before naming the property.
- *3 groups of 4 and 4 groups of 3 are different facts that must be memorized separately.*
  Show the same array as both 'read by rows' and 'read by columns' descriptions before introducing the word commutative.

Read together FOR THE STUDENT

In 3 x 4 = 12, the numbers 3 and 4 are factors. They are the numbers you multiply. The answer, 12, is the product. In an array, count the rows and columns. Rows and columns are the factors. The total number of dots is the product. It does not matter which factor is bigger. Both numbers are still factors.

Going further: Every multiplication equation has two factors and one product. Factors are the numbers being multiplied. The product is the answer. In 5 x 2 = 10, 5 and 2 are factors. 10 is the product. In an array, rows and columns show the factors. The total dots show the product. Turning an array does not change its product. The dot count stays the same.

Watch it work

Name the factors and product in 3 x 4 = 12.

1. Find the multiplication sign. It splits the two factors.
2. 3 and 4 are factors. They are being multiplied.
3. 12 is the product. It is the answer.
4. Factors: 3 and 4. Product: 12.

The idea: The numbers before the equals sign are factors. The answer is the product.

An array has 2 rows and 5 columns. Name the factors and product.

1. Count the rows: 2. That is one factor.
2. Count the columns: 5. That is the other factor.
3. Count all the dots: 10. That is the product.
4. Write it as an equation: 2 x 5 = 10.

The idea: Rows and columns in an array are the two factors. The total count is the product.

Practice: Factor and product vocabulary

1. In 4 x 5 = 20, which numbers are the factors?

(A) 20 only (B) 4 and 5 (C) 20 and 4

2. In 6 x 3 = 18, what is the product?

(A) 3 (B) 6 (C) 18

3. This array has 3 rows and 5 columns, 15 dots total. What are the factors?

(A) 15 only (B) 15 and 3 (C) 3 and 5

4. This array has 4 rows and 2 columns, 8 dots total. What is the product?

(A) 2 (B) 4 (C) 8

5. This array was turned. It now has 5 rows and 3 columns. Before, it had 3 rows and 5 columns. What is the product now?

(A) 8 (B) It changed, so you must recount (C) 15

6. A student says 3 x 4 and 4 x 3 are two different facts. Are they?

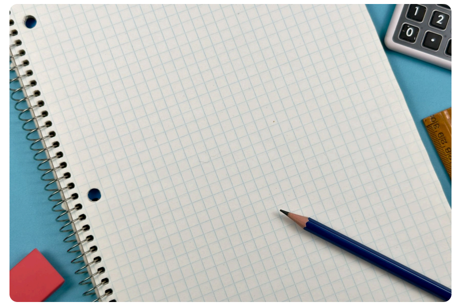
(A) Yes, because one number is bigger first

(B) No, both equal 12, just with factors swapped

(C) Yes, because the group sizes are different

MATH · UNIT 1 · LESSON 4

Commutative property shown by rotating an array



Goal: Given an array, the student rotates it 90 degrees (physically or by redrawing) and explains why the total does not change.

About 18 minutes

Teach it FOR THE PARENT

1. **Say:** Grab any array you built this week, like 3 rows of 4 dots. Turn your paper sideways. What do you notice about the dots?
   Look for: Student notices the dots look arranged differently but nothing was added or erased.
2. **Say:** An array has rows and columns. When you rotate it 90 degrees, rows become columns and columns become rows. But you do not add or remove any dots. So the total count cannot change.
   Look for: Student can restate that rotation changes the view, not the count.
3. **Say:** Watch me. I draw 3 rows of 4 dots, that's 12. I turn my paper. Now I see 4 rows of 3. Let's count: still 12! Why didn't the total change?
   Look for: Student says the same dots are there, just arranged a new way.
4. **Say:** Quick check: if 6 rows of 5 dots gives 30 dots, how many dots are in 5 rows of 6, after rotating? Why?
   Look for: Student answers 30, and says rotating did not change the dot count.
5. **Say:** Your turn. Draw 2 rows of 7 dots. Rotate your paper 90 degrees. What array do you see now? What is the total both times?
   Look for: Student draws 7 rows of 2, states 14 dots both times, and explains why.
6. **Say:** You just showed that turning an array never changes the total. That's why swapped fact pairs, like 3x4 and 4x3, always match. Let's practice this idea.
   Look for: Student is ready to apply the rotation idea to new fact pairs in practice.

If they get stuck

- *Multiplying always makes a number bigger.*
  On Day 14 explicitly flag: this is true for whole numbers greater than 1, but not always true once fractions arrive - do not resolve it fully, just name the boundary.
- *Rotating an array changes how many dots there are, because it looks different.*
  Have students physically rotate a card of counters and count before and after, side by side, several times before naming the property.
- *3 groups of 4 and 4 groups of 3 are different facts that must be memorized separately.*
  Show the same array as both 'read by rows' and 'read by columns' descriptions before introducing the word commutative.

Read together FOR THE STUDENT

Make an array of dots with rows and columns, like 3 rows of 4. Turn your paper sideways. Now you see 4 rows of 3. Count the dots. Same number! Turning the array does not add or remove any dots. It just changes which way you look at it. This is why 3 x 4 and 4 x 3 give the same answer.

Going further: An array has rows (going across) and columns (going down). A 3-by-4 array has 3 rows of 4. Turn it 90 degrees. Now it is a 4-by-3 array, 4 rows of 3. The dots did not change. Only your view changed. So 3 x 4 = 4 x 3. This works for any array. Turning it never changes the total count.

Watch it work

Build an array with 3 rows of 4 dots. Rotate it 90 degrees. Does the total change?

1. Draw 3 rows of 4 dots. That is 3 x 4 = 12 dots.
2. Turn the paper 90 degrees. The rows are now columns.
3. Count again: now you see 4 rows of 3 dots.
4. 4 x 3 also equals 12. No dots were added or removed.

The idea: Turning an array only changes how you view it, not how many dots it has.

Build an array with 5 rows of 2 dots. Rotate it 90 degrees. What array do you see, and what is the total?

1. Draw 5 rows of 2 dots. That is 5 x 2 = 10 dots.
2. Turn the paper 90 degrees to rotate the array.
3. Now you see 2 rows of 5 dots.
4. 2 x 5 also equals 10. The total stayed the same.

The idea: Rotating swaps rows and columns, but the same dots are still there.

Practice: Commutative property shown by rotating an array

1. An array has 3 rows of 5 dots. You rotate it 90 degrees. How many dots are there now?

- (A) 8 dots, because rows and columns add
- (B) It changes, so you must count again
- (C) 5 dots, because that's one row
- (D) 15 dots, same as before

2. You rotate a 4-by-2 array 90 degrees. What array do you see now?

- (A) 8 rows of 1
- (B) 2 rows of 4
- (C) 4 rows of 2, unchanged
- (D) 6 rows of 1

3. A 2-by-6 array has 12 dots. After rotating 90 degrees, how many dots are there?

- (A) 6 dots, because one side shrinks
- (B) It depends, so you must recount
- (C) 12 dots
- (D) 24 dots, because it doubles

4. Mia says 7 x 3 and 3 x 7 must be different facts. Is she right?

- (A) No, but only for small numbers like 2 and 3
- (B) Yes, group size and group count are always fixed
- (C) Yes, because 7 groups feels bigger than 3 groups
- (D) No, rotating the array shows they match

5. An array has 6 rows of 8 dots, which is 48 dots. Rotate it 90 degrees. What is the new total?

- (A) More than 48, because 8 is bigger than 6
- (B) You need to count all 48 dots again
- (C) 14 dots, adding rows and columns
- (D) 48 dots

6. Which statement best explains why rotating an array never changes the total?

- (A) Rows always have more dots in them than columns do, so the total stays
- (B) Turning it makes each dot count as less
- (C) No dots are added or taken away when you turn it
- (D) You have to recount to be sure

Equal-groups and array pictures as two representations of one fact

Goal: The student compares an equal-groups picture and an array for the same fact and explains what stays the same and what looks different.

About 18 minutes

Teach it FOR THE PARENT

1. **Say:** Look at these two pictures. One shows 3 plates with 4 cookies each. One shows cookies lined up in 3 rows of 4. Do they show the same number of cookies?
   Look for: Student guesses yes or is unsure, and looks ready to check by counting.
2. **Say:** Both pictures can show 3 x 4. The plates picture spreads groups apart. The array lines groups into neat rows and columns. The groups, the group size, and the total all stay the same. Only the arrangement changes.
   Look for: Student can repeat that arrangement changed but total, groups, and group size did not.
3. **Say:** Let's compare 4 groups of 2 counters with an array of 4 rows and 2 columns. Count the groups in each picture. Count how many are in each group. Count the total in each picture.
   Look for: Student counts 4 groups, 2 per group, 12 total in both pictures... total should be 8, matches both.
4. **Say:** Same question, new numbers. I rotate an array of 3 rows and 5 columns to make 5 rows and 3 columns. Does the total change?
   Look for: Student says no, the total stays 15, only rows and columns swapped.
5. **Say:** Here is a fresh pair: 5 groups of 3 dots, and an array of 5 rows, 3 columns. Tell me what is the same and what looks different between them.
   Look for: Student names same total (15), same groups (5), same group size (3), different only in layout.
6. **Say:** You just compared two pictures of the same fact. Now try more comparisons on your own in the practice set.
   Look for: Student moves into independent items ready to compare pictures, not just count them.

If they get stuck

- *Multiplying always makes a number bigger.*
  On Day 14 explicitly flag: this is true for whole numbers greater than 1, but not always true once fractions arrive - do not resolve it fully, just name the boundary.
- *Rotating an array changes how many dots there are, because it looks different.*
  Have students physically rotate a card of counters and count before and after, side by side, several times before naming the property.
- *3 groups of 4 and 4 groups of 3 are different facts that must be memorized separately.*
  Show the same array as both 'read by rows' and 'read by columns' descriptions before introducing the word commutative.

Read together FOR THE STUDENT

An equal-groups picture shows separate clusters, like 3 plates with 4 cookies each. An array lines up the same cookies in neat rows and columns, like 3 rows of 4. Both show 3 x 4 = 12. Nothing was added or taken away. The cookies just got organized into a grid. Same total, same groups, same size per group. Only the arrangement on the page changed.

Going further: Equal-groups pictures show separate clusters. Arrays show rows and columns. Both can show the same fact, like 3 x 4. The number of groups stays the same. The size of each group stays the same. The total stays the same. Only the layout changes. An array also lets you read it by rows (3 groups of 4) or by columns (4 groups of 3). Both readings give 12, showing 3 x 4 = 4 x 3.

Watch it work

Compare: 3 plates with 4 apples each, and an array of 3 rows and 4 columns of apples.

1. Count the plates picture: 3 groups, 4 apples in each group.
2. Count the array: 3 rows, 4 apples in each row.
3. Both pictures show 12 apples total. The total did not change.
4. The plates are spread apart. The array is lined up in a grid. That is the only difference.

The idea: Same groups, same size, same total, only the arrangement on the page is different.

An array has 4 rows and 3 columns. Read it two ways.

1. Read by rows: 4 groups of 3. That is 4 x 3.
2. Read by columns: 3 groups of 4. That is 3 x 4.
3. Count all dots once: 12 dots, no matter which way you read it.
4. 4 x 3 and 3 x 4 both equal 12, because it is the same array.

The idea: One array can be read two ways, and both readings name the same total.

Practice: Equal-groups and array pictures as two representations of one fact

1. One picture shows 3 groups of 5 stars. Another shows an array with 3 rows and 5 columns of stars. What is the total in each picture?

(A) 18 in both (B) 15 in both (C) 15 in the groups picture, 8 in the array

2. An array has 4 rows and 2 columns of dots. You turn the array so it has 2 rows and 4 columns. How many dots are there now?

(A) Fewer dots, because it looks smaller (B) More dots, because it looks wider

(C) The same number as before

3. Picture A shows 4 bags with 3 apples each, spread apart. Picture B shows an array with 4 rows and 3 columns of apples. What is different between the pictures?

(A) The total number of apples (B) How many apples are in each group

(C) Only how the apples are arranged on the page

4. An array has 5 rows and 2 columns. Read it by rows, then by columns. What two facts do you get?

(A) 10 x 1, because there is only one array (B) 5 x 2 and 2 x 5

(C) 5 x 2 only, because columns do not count as groups

5. Maya says 3 groups of 6 and 6 groups of 3 are two different facts. Is she right?

(A) Yes, because the group sizes are switched, so they are two different facts

(B) No, both give the same total, just arranged differently

(C) Yes, because one picture has more groups in it than the other one does

6. An equal-groups picture shows 4 groups of 5 dots, spread apart. An array shows 4 rows and 5 columns of dots. What stays the same between the two pictures?

(A) Only the total stays the same (B) Nothing stays the same, they are different pictures

(C) The number of groups, the size of each group, and the total

MATH · UNIT 1 · LESSON 6

Equal-groups word problems within 100

Goal: Given a word problem describing equal groups, the student selects and draws the matching picture and writes the equation.

About 18 minutes

Teach it FOR THE PARENT

1. **Say:** A word problem is like a tiny mystery. It hides two numbers inside a story. Today you find those two numbers. Then you draw them and turn them into one equation.
   Look for: Student is ready to listen for two hidden numbers in a story.
2. **Say:** Listen: 'A baker makes 6 trays. Each tray has 4 muffins.' I draw 6 circles. I put 4 dots in each circle. I count all the dots: 24. I write 6 x 4 = 24. My picture matches the story exactly.
   Look for: Student notices the drawing has the same groups and group size named in the story.
3. **Say:** Read this problem: 'A garden has 5 rows. Each row has 3 flowers.' Which number is the group count? Which number is the group size?
   Look for: Student names 5 as the group count and 3 as the group size, matching their roles in the story.
4. **Say:** Now draw the picture for the garden problem. Draw 5 groups. Put 3 flowers, dots, or Xs in each group.
   Look for: Drawing shows exactly 5 groups, each with exactly 3 items, matching the story order.
5. **Say:** Count every item in your picture. Then write the equation for the garden problem.
   Look for: Total is 15, and the equation is 5 x 3 = 15, groups written before group size.
6. **Say:** Here is a new problem: 'A shelf holds 4 boxes. Each box holds 7 books.' Draw the picture and write the equation.
   Look for: Student draws 4 groups of 7 and writes 4 x 7 = 28 without copying the garden numbers.
7. **Say:** You just turned two stories into pictures and equations. Every practice problem below works the same way. Find the groups. Find the group size. Draw it. Then write the equation.
   Look for: Student is ready to repeat the same three steps on new numbers in practice.

If they get stuck

- *Multiplying always makes a number bigger.*
  On Day 14 explicitly flag: this is true for whole numbers greater than 1, but not always true once fractions arrive - do not resolve it fully, just name the boundary.
- *Rotating an array changes how many dots there are, because it looks different.*
  Have students physically rotate a card of counters and count before and after, side by side, several times before naming the property.
- *3 groups of 4 and 4 groups of 3 are different facts that must be memorized separately.*
  Show the same array as both 'read by rows' and 'read by columns' descriptions before introducing the word commutative.

Read together FOR THE STUDENT

A word problem might say '5 bags of 3 apples.' That means 5 equal groups. Each group has 3 apples. Draw 5 circles. Put 3 dots in each circle. Count the total dots. Then write the equation: 5 x 3 = 15. The picture and the equation say the same thing two ways.

Going further: An equal-groups problem names two numbers: how many groups, and how many in each group. Find both numbers first. Draw that many groups, with that many objects in each. The picture must match the story exactly, not just have the right total. The equation is groups x group size = total. Writing 5 x 3 = 15 records what the picture shows.

Watch it work

A farmer packs 4 baskets. Each basket has 6 eggs. How many eggs in all?

1. Find the group count: 4 baskets. This is the number of groups.
2. Find the group size: 6 eggs. This is how many in each group.
3. Draw 4 circles. Put 6 dots inside each circle.
4. Count all the dots: 24 dots in all.
5. Write the equation: 4 x 6 = 24.

The idea: The picture shows groups first; the equation just records what you drew.

A classroom has 3 rows of desks. Each row has 5 desks. How many desks in all?

1. Find the group count: 3 rows. Rows act like groups.
2. Find the group size: 5 desks in each row.
3. Draw an array: 3 rows, 5 desks in each row.
4. Count all desks: 15 desks in all.
5. Write the equation: 3 x 5 = 15.

The idea: An array is just equal groups lined up in neat rows and columns.

Practice: Equal-groups word problems within 100

1. A pet store has 3 tanks. Each tank has 4 fish. Which picture matches this story?

(A) 3 groups of 4 fish (B) 7 fish in one tank (C) 4 groups of 3 fish

2. A baker makes 5 trays. Each tray has 6 muffins. What is the group size?

(A) 5 (B) 11 (C) 6

3. A parking lot has 4 rows of cars. Each row has 5 cars. What equation matches?

(A) 4 + 5 = 9 (B) 5 x 4 = 20 (C) 4 x 5 = 20

4. A farmer has 6 baskets. Each basket has 3 apples. How many apples in all?

(A) 3 apples (B) 9 apples (C) 18 apples

5. An array has 4 rows and 5 columns of chairs. A helper turns it sideways so it looks like 5 rows and 4 columns. How many chairs are there now?

(A) 20 chairs, same as before (B) 25 chairs, because it looks bigger

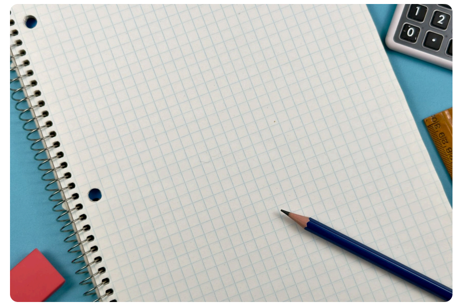
(C) 9 chairs, because the shape changed

6. A toy store has 7 shelves. Each shelf holds 4 robots. Which picture and equation match this story?

(A) 7 groups of 4, equation 7 + 4 = 11 (B) 4 groups of 7, equation 4 x 7 = 28

(C) 7 groups of 4, equation 7 x 4 = 28

Limits of the repeated-addition analogy for multiplication



Goal: The student identifies a case where repeated addition describes a situation but an array does not fit naturally (e.g., 3 groups of a half-sandwich), and explains why.

About 18 minutes

Teach it FOR THE PARENT

1. **Say:** 3 friends each eat half a sandwich. How much sandwich did they eat all together? Can you picture rows and columns for this, like we did for whole cookies?
   Look for: Student notices adding halves is easy but drawing neat rows/columns feels odd or impossible.
2. **Say:** Repeated addition just adds the same number again and again. It does not care if that number is a whole thing or a fraction. An array needs whole, same-size units lined up in rows and columns. That is the difference we are hunting for today.
   Look for: Student can restate that addition only needs a repeated amount, arrays need whole matching units.
3. **Say:** Watch: 3 shelves, 4 books each. 4+4+4=12. Now I draw 3 rows of 4 books, all whole, all the same size. Does the array fit here? Why?
   Look for: Student says yes, because each book is a whole matching unit that lines up evenly.
4. **Say:** Now: 3 kids, each with 1/2 a sandwich. 1/2+1/2+1/2=3/2. Try to draw an array of half-sandwiches in rows and columns. What goes wrong?
   Look for: Student says the halves are not whole matching units, so rows and columns do not line up naturally.
5. **Say:** New case: 4 friends each get 1/4 of a pizza. Does repeated addition work? Does an array fit naturally? Tell me why or why not, in your own words.
   Look for: Student adds 1/4 four times correctly, then explains array does not fit because 1/4 pizza is not a whole matching unit.
6. **Say:** You just found a case where one multiplication picture works and the other does not. Now try some practice problems that test this same idea in new situations.
   Look for: Student is ready to apply the addition-works-array-does-not idea to new, unseen contexts.

If they get stuck

- *Multiplying always makes a number bigger.*
  On Day 14 explicitly flag: this is true for whole numbers greater than 1, but not always true once fractions arrive - do not resolve it fully, just name the boundary.
- *Rotating an array changes how many dots there are, because it looks different.*
  Have students physically rotate a card of counters and count before and after, side by side, several times before naming the property.
- *3 groups of 4 and 4 groups of 3 are different facts that must be memorized separately.*
  Show the same array as both 'read by rows' and 'read by columns' descriptions before introducing the word commutative.

Read together FOR THE STUDENT

Multiplication can mean groups that are the same size, like 3 bags with 4 apples each. You can add 4+4+4 or draw an array with rows and columns. But some groups are not whole things. 3 people each eat half a sandwich. You can still add 1/2+1/2+1/2. But you cannot draw neat rows and columns of half-sandwiches, because arrays need whole, same-shaped squares. Adding still works. The array picture does not fit.

Going further: Repeated addition and arrays both model equal groups. Repeated addition just adds the same number again and again, like 1/2+1/2+1/2. Arrays need whole units arranged in even rows and columns. When the group size is a fraction, like half a sandwich, addition still works, but an array has no clean rows and columns to draw. The array model breaks down for fractional group sizes even though the addition model still describes the situation correctly.

Watch it work

3 shelves each hold 4 books. Show this with repeated addition AND an array. Do both fit?

1. Repeated addition: 4 + 4 + 4 = 12, one 4 for each shelf.
2. Array: draw 3 rows, 4 books in each row, all books whole and same size.
3. Check the array: every row has 4 whole books, rows line up evenly.
4. Both models fit, because the group size (4 books) is a whole, countable thing.

The idea: When groups are whole, same-size objects, repeated addition and arrays both work.

3 kids each get 1/2 of a sandwich. Show this with repeated addition. Try an array. Do both fit?

1. Repeated addition: $1/2 + 1/2 + 1/2 = 3/2$, one half for each kid.
2. Try an array: try to make 3 rows with columns of 'half-sandwiches'.
3. Check the array: a half-sandwich is not a whole unit, so rows and columns will not line up evenly like whole objects do.
4. Repeated addition still describes the situation, but the array does not fit, because the group size is not a whole, uniform unit.

The idea: Repeated addition works with fraction-size groups; an array needs whole, uniform units, so it can fail even when addition still works.

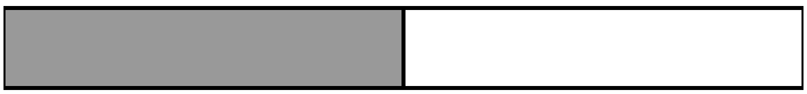
Name: ________________

Practice: Limits of the repeated-addition analogy for multiplication

1. 3 boxes each have 5 whole apples. Does an array fit this situation?

- (A) Yes, because each apple is a whole, same-size unit that lines up in rows and columns.
- (B) No, because multiplying always makes numbers bigger.
- (C) No, because rotating the array would change the total.

2. 2 kids each get 1/2 of a granola bar. Can you add to find the total amount?



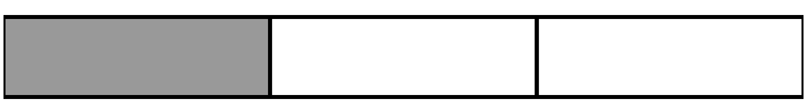
1/2 of a granola bar

- (A) No, you can only add whole numbers.
- (B) No, because an array would not fit, so addition will not either.
- (C) Yes, 1/2 + 1/2 = 1 whole bar.

3. 4 rows of 3 stars and 3 rows of 4 stars. Are these two different facts, or the same total?

- (A) Different facts, because 4 groups of 3 and 3 groups of 4 must be memorized separately.
- (B) The same total, because turning the array does not change how many stars there are.
- (C) Different totals, because turning an array changes how many dots are in it.

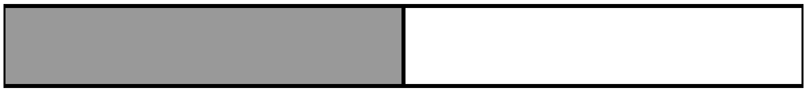
4. 3 groups of 1/3 pizza each. Does repeated addition describe this? Does an array fit naturally?



1/3 of a pizza

- (A) Neither addition nor an array works here.
- (B) Both addition and an array work perfectly here.
- (C) Addition works (1/3+1/3+1/3=1), but an array does not fit naturally because 1/3 pizza is not a whole matching unit.

5. 5 friends each drink 1/2 cup of juice. Which statement is true?



1/2 cup, repeated 5 times

- Ⓐ 1/2+1/2+1/2+1/2+1/2 works to find the total, but drawing a neat array of half-cups does not fit naturally.
- Ⓑ Neither method works here, because 1/2 is too small a number to multiply or add, so the friends' juice cannot be totaled at all.
- Ⓒ An array works fine here, just like it does with 5 groups of whole cups, because half-cups line up in rows and columns the same way.

6. 6 groups of 2 cookies. Someone says an array cannot fit because 6 rows is too many. Are they right?

- Ⓐ No, an array of 6 rows and 2 columns fits fine, because cookies are whole matching units.
- Ⓑ Yes, because too many rows always breaks an array, and 6 rows is past the limit for a picture of cookies to make sense.
- Ⓒ Yes, because 6 groups of 2 cookies and 2 groups of 6 cookies are two different facts, so only one of them can fit an array.

MATH · UNIT 1 · LESSON 8

Commutative property as evidenced by array rotation

Goal: Given a multiplication fact never shown in class, the student draws an array, rotates it, and explains using the picture why the two factor orders give the same product.

About 18 minutes

Teach it FOR THE PARENT

1. **Say:** Quick question: do you think 7 x 4 and 4 x 7 give the same answer, or different answers? Don't calculate yet, just guess.
   Look for: A guess, either way, that the student is willing to test with a picture.
2. **Say:** An array is dots in rows and columns. When you rotate an array a quarter turn, the rows and columns swap. But every single dot is still there. Nothing got added, nothing got erased.
   Look for: Student can restate that rotation moves dots but does not change how many there are.
3. **Say:** Watch: I draw 5 rows of 3 dots, that's 15 dots. I rotate it. Now it's 3 rows of 5 dots. I count again: still 15. So 5 x 3 = 3 x 5. Does the total ever change when I rotate?
   Look for: Student says no, the total stays the same, only the arrangement changes.
4. **Say:** Your turn. Draw an array for 6 x 2. Rotate it in your head or on paper. What array do you get, and how many dots are there now?
   Look for: Student draws 6 rows of 2, rotates to 2 rows of 6, and reports 12 dots both times.
5. **Say:** Now try a fact you have not drawn before: 4 x 6. Draw the array, rotate it, and explain in your own words why 4 x 6 equals 6 x 4.
   Look for: Student draws a fresh 4-by-6 array, rotates it to 6-by-4, and explains using dot count, not memorized facts.
6. **Say:** You just proved a multiplication fact with a picture instead of memorizing it. Now you'll do this for facts you've never seen before. Use the array and rotation to explain, every time.
   Look for: Student is ready to apply array rotation reasoning to new, unseen fact pairs.

If they get stuck

- *Multiplying always makes a number bigger.*
  On Day 14 explicitly flag: this is true for whole numbers greater than 1, but not always true once fractions arrive - do not resolve it fully, just name the boundary.
- *Rotating an array changes how many dots there are, because it looks different.*
  Have students physically rotate a card of counters and count before and after, side by side, several times before naming the property.
- *3 groups of 4 and 4 groups of 3 are different facts that must be memorized separately.*
  Show the same array as both 'read by rows' and 'read by columns' descriptions before introducing the word commutative.

Read together FOR THE STUDENT

An array is dots in rows and columns. Turn it sideways and you get a new array. Count the dots. Same number! 3 rows of 4 turned sideways becomes 4 rows of 3. Nothing fell off, nothing got added. That is why 3 x 4 and 4 x 3 give the same answer. The dots just moved, they did not change.

Going further: A 3-by-4 array has 3 rows and 4 columns, total 12 dots. Rotate it a quarter turn: now it has 4 rows and 3 columns, still 12 dots. Rows and columns swapped, but total dots did not change. This shows 3 x 4 = 4 x 3. This works for any two factors, because rotating never adds or removes dots, only changes which way you're counting them.

Watch it work

Show why 5 x 3 = 3 x 5 using an array.

1. Draw 5 rows of 3 dots. Count all dots: 15 dots total, because 5 groups of 3 is 15.
2. Rotate the array a quarter turn. Now count rows and columns again: 3 rows, 5 columns.
3. Count the dots again: still 15. Rotating only turned the picture, it did not add or remove dots.
4. So 5 x 3 = 15 and 3 x 5 = 15. Same array, same dots, same answer.

The idea: Rotating an array changes rows and columns but never changes the total number of dots.

Show why 6 x 2 = 2 x 6 using an array.

1. Draw 6 rows of 2 dots. Count: 6 groups of 2 is 12 dots total.
2. Rotate the array a quarter turn. Now it shows 2 rows of 6 dots.
3. Count the rotated array: 2 groups of 6 is still 12 dots. No dots were added or taken away.
4. 6 x 2 and 2 x 6 both equal 12, because rotation only changes the arrangement, not the count.

The idea: The same set of dots can be seen as 6 groups of 2 or 2 groups of 6, the picture proves both counts match.

Practice: Commutative property as evidenced by array rotation

1. An array has 4 rows of 5 dots. You rotate it. How many dots are in the rotated array?

(A) You must recount, it could be a different number (B) 20 dots, same as before

(C) More than 20 dots

2. A 3-by-6 array is rotated. What does the new array look like?

(A) 9 rows of 2 dots (B) 3 rows of 6 dots, unchanged (C) 6 rows of 3 dots

3. Which equation does a rotated 4-by-7 array prove true?

(A) 4 x 7 = 7 x 4 (B) 4 x 7 is bigger after rotating (C) 4 x 7 = 4 + 7

4. Mia says 3 groups of 8 and 8 groups of 3 are two different facts to memorize. What does the array picture show?

(A) Mia is right, they must be memorized separately

(B) They are the same total, just rotated (C) They are different because 8 is bigger than 3

5. You rotate a 5-by-9 array. A friend says the new array must have more dots because it looks bigger. What is wrong with that idea?

(A) Rotating never adds dots, only turns the picture

(B) Only some rotations keep the same total

(C) The friend is correct, rotated arrays gain dots

6. Which explanation best proves 6 x 4 = 4 x 6 using a picture?

(A) 6 x 4 must be checked separately from 4 x 6 by recounting from scratch

(B) Both facts equal 24 because that's what the times table says

(C) A 6-by-4 array rotated becomes a 4-by-6 array with the same 24 dots

Answer key

Math

Lesson 1	1. C 2. A 3. B 4. C 5. A 6. C
Lesson 2	1. A 2. A 3. A 4. C 5. A 6. A
Lesson 3	1. B 2. C 3. C 4. C 5. C 6. B
Lesson 4	1. D 2. B 3. C 4. D 5. D 6. C
Lesson 5	1. B 2. C 3. C 4. B 5. B 6. C
Lesson 6	1. A 2. C 3. C 4. C 5. A 6. C
Lesson 7	1. A 2. C 3. B 4. C 5. A 6. A
Lesson 8	1. B 2. C 3. A 4. B 5. A 6. C
Unit 1 check-up	1. D 2. C 3. C 4. B 5. B 6. A 7. D 8. A 9. B 10. C 11. A 12. A 13. C 14. B 15. B 16. A

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