CLAREZA ACADEMY

Grade 9 -- Mathematics Math

Free sample: Unit 1, Lessons 1 to 10

These are 10 lessons from *Clareza Grade 9: Mathematics*, a ninth grade year a parent runs at the kitchen table. Each lesson is a short script for you, a page to read together, and a practice sheet you may copy for your household. Answers are at the back.

Clareza Grade 9: Mathematics. Sample edition.

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Clareza is a secular curriculum. Standards references: Common Core State Standards and the Next Generation Science Standards.



MATH · UNIT 1

Linear Relationships as Rules, Graphs, and Stories

This is the unit everything else depends on. Your child learns that a linear relationship, a starting amount plus a steady rate of change, can be shown as a table, a graph, an equation, or a story about a phone plan or a tank of water, and that these are four views of the same thing, not four different skills. They also meet slope, intercepts, domain/range, and function notation for the first time.

In this unit

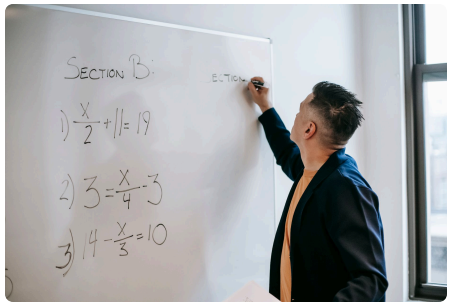
1. Starting value and rate of change
2. Computing slope from two points
3. Slope is the same everywhere
4. Which representation came first
5. Writing equations in slope-intercept form
6. Function notation and what it means
7. Domain and range in context
8. When two plans cost the same
9. Choosing when to use linear models
10. Slope and intercept vocabulary

MATH · UNIT 1 · LESSON 1

Starting value and rate of change

Goal: Given a real-world description of a linear situation (e.g., a text-messaging plan), the student identifies the starting amount and the constant amount of change per unit and states which is which.

About 40 minutes



Teach it FOR THE PARENT

1. **Say:** Picture your phone has 500 MB of data saved up right now, and every video you watch uses 40 MB more. Before you watch anything, you already have 500 MB, that number doesn't move. But every video costs you 40 MB, over and over. Today we're just learning to spot those two different kinds of numbers in a story.
   Look for: Student can say which number (500 or 40) is already there and which one repeats.

2. **Say:** Every one of these stories has a starting amount, what's true before anything happens, and a constant amount of change, the same number added or taken away for every single unit. The change number is almost always glued to a word like 'per', 'each', or 'every'. The starting number is the one just sitting there with no unit word stuck to it.
   Look for: Student can state the two-part rule in their own words: one number is 'before anything', one number 'repeats per unit'.

3. **Say:** Let's read the moving company example together: a moving company charges a flat fee of $50 just to show up, plus $25 for every hour the movers work. Find the number with no 'per' word attached. That's our starting amount. Now find the number attached to 'for every hour'. That's our constant change. Let's check it: if 0 hours were worked, would the cost just be the flat fee? Yes, so we know we picked correctly.
   Look for: Student follows each step: names the flat, unit-free number as the starting amount; names the 'per hour' number as the change; confirms using the hour = 0 check.

4. **Say:** Your turn, fresh story: A parking garage charges a $6 entry fee, plus $2 for every hour you park. What's the starting amount, and what's the constant amount of change per unit? Say which is which, and check your answer using the 'what if 0 hours' test.
   Look for: Student identifies $6 as the starting amount (present at 0 hours) and $2 per hour as the constant change, and explicitly runs the 0-hour check.

5. **Say:** You've got the two questions to ask every time: what's true before anything happens, and what number repeats for every unit? Now use those two questions on the practice questions below, each one is a new story, so you'll have to hunt for the same clues in a new setting every time.
   Look for: Student is ready to apply the same two questions (before-anything number, per-unit number) to unfamiliar contexts in the practice set.

If they get stuck

- *The equals sign means 'now write down the answer' rather than 'these two things are the same value', so an equation like 3 + 4 = 7 feels fine but 7 = 3 + 4 looks 'backwards', and an equation with expressions on both sides (used to find where two plans cost the same) is confusing because there's nothing to 'work out' on one side.*
  In the equation-writing and intersection lessons, deliberately present equations in reflexive and operations-on-both-sides form from the first worked example (not just a-op-b=c), and have the student verbally restate what each side represents in the context before manipulating symbols.
- *In an expression like '3b', the letter b is a label for the object itself ('3 buns') rather than a variable standing for a number (the number of buns), so combining or comparing terms with letters gets treated like combining objects rather than combining quantities.*
  In the equation-writing and function-notation lessons, require the student to state the units of every variable and every term aloud or in writing before symbols are manipulated ('m is a number of months, so 15m is a number of DOLLARS, not months'), do this as a fixed step, not a one-time warning.
- *When translating a comparison sentence like 'there are six times as many minutes used by plan A as by plan B' into symbols, the student writes the equation in the order the words appear (6A = B) rather than matching quantities correctly (A = 6B).*
  Teach a check-with-a-number step as a fixed routine: pick a small value for one quantity, compute what the other should be from the sentence, then test both candidate equations against that pair before committing.

Read together FOR THE STUDENT

A linear situation has two parts: a starting amount (what you have before anything happens) and a constant change (how much it goes up or down every single time, by the same amount). Think of a video game: you start with 200 coins already in your pocket, and you earn 10 more coins every level. The 200 is your starting amount, it's there at level 0, before you've done anything. The 10 is the constant amount of change, it's the same every level, so you can predict it. When you read a word problem, find the number attached to 'per', 'each', or 'every', that's the change. The number that's just sitting there with no 'per' attached, that's the start.

Going further: Every linear situation described in words has a starting value (the amount present before any units of the input have occurred, at time zero, item zero, month zero) and a constant rate of change (the fixed amount added or subtracted for each additional unit). In the sentence itself, the rate of change is almost always tied to a 'per unit' phrase: 'per month', 'for every text', 'each mile'. The starting value is the leftover number with no unit attached to it, it's what's true before any of that per-unit activity happens. In y = mx + b form, b is the starting value and m is the constant rate of change, but at this stage you're identifying these two quantities directly from the words, not yet writing the equation.

Watch it work

A moving company charges a flat fee of $50 just to show up, plus $25 for every hour the movers work. Identify the starting amount and the constant amount of change per unit, and state which is which.

1. Find the number that has no 'per unit' language attached to it: $50 is called a 'flat fee' and is charged 'just to show up', meaning it applies even if the movers work zero hours. That makes $50 the starting amount.
2. Find the number attached to 'per unit' language: $25 is tied to the phrase 'for every hour', this means $25 gets added again and again, once for each additional hour. That makes $25 the constant amount of change.
3. Check by testing hour = 0: if the movers work 0 hours, the cost should just be the flat fee, $50. This matches the starting amount, confirming the identification is correct.
4. State the answer clearly and label each part: starting amount = $50 (present before any hours are worked); constant amount of change = $25 per hour (added for each additional hour).

The idea: The starting amount is whatever is true when the input is zero; the constant amount of change is whatever number is glued to 'per', 'each', or 'every' in the sentence.

A tank already holds 8 gallons of water when a hose is turned on, and the hose adds 3 gallons of water every minute. Identify the starting amount and the constant amount of change per unit, and state which is which.

1. Locate the unit-free quantity that exists before the action starts: the tank 'already holds 8 gallons' before the hose is turned on, before any minutes have passed. That makes 8 gallons the starting amount.
2. Locate the quantity tied to a 'per unit' phrase: '3 gallons of water every minute' ties 3 to each minute that passes. That makes 3 gallons per minute the constant amount of change.
3. Check by testing minute = 0: at the instant the hose is turned on, before a minute has passed, the tank should hold exactly the starting amount, 8 gallons, this matches.
4. State the answer clearly: starting amount = 8 gallons (in the tank before the hose runs); constant amount of change = 3 gallons per minute (added for every minute the hose runs).

The idea: Testing the situation at input = 0 always isolates the starting amount, because every per-unit change contributes nothing yet at that instant.

Practice: Starting value and rate of change

1. A candle is 12 inches tall when it is lit, and it burns down 0.5 inches every hour. What is the starting amount, and what is the constant amount of change per unit?

- **A** Starting amount = 0.5 inches; constant change = 0.5 inches per hour
- **B** Starting amount = 12 inches; constant change = 0.5 inches per hour
- **C** Starting amount = 12 inches; constant change = 12 inches per hour
- **D** Starting amount = 0.5 inches; constant change = 12 inches per hour

2. A gym membership costs $20 to join, plus $15 for every month. What is the starting amount, and what is the constant amount of change per unit?

- **A** Starting amount = $20; constant change = $15 per month
- **B** Starting amount = $20; constant change = $35 per month
- **C** Starting amount = $15; constant change = $20 per month
- **D** Starting amount = $35; constant change = $15 per month

3. A hiking trail starts at an elevation of 400 feet, and the trail rises 60 feet for every mile you walk. What is the starting amount, and what is the constant amount of change per unit?

- **A** Starting amount = 460 feet; constant change = 60 feet per mile
- **B** Starting amount = 400 feet; constant change = 60 feet per mile
- **C** Starting amount = 400 feet; constant change = 460 feet per mile
- **D** Starting amount = 60 feet; constant change = 400 feet per mile

4. A writer has already written 1,200 words of a story, and plans to write 300 more words every day. A friend says: 'the equation should say the total is 300 words, d days.' The friend writes: 300 = d. What mistake is being made?

(A) The friend is treating '=' as 'here comes the answer' instead of as 'these two amounts are equal', so the equation doesn't actually represent the relationship between days and words written

(B) The friend used 'd' to mean 'a day itself' instead of 'the number of days', which is a different error than the one actually happening here, since the letter is standing for the wrong kind of thing rather than the equals sign being misused

(C) The friend reversed the order of a comparison sentence, writing the two quantities in the wrong order on either side of the equals sign, so the fix is simply to swap the sides and write d = 300 instead, which then represents the story correctly

(D) The friend correctly identified the constant change, so there's no mistake to point out

5. A well already holds 250 gallons of water, and a pump adds 40 gallons every minute. Which number is the constant amount of change per unit, and how do you know?

(A) 250 gallons, because it is the amount already in the well before the pump starts

(B) 40 gallons, because it is tied to the phrase 'every minute', meaning it's added again for each additional minute

(C) 40 gallons, because it is the smaller of the two numbers

(D) 250 gallons, because it is the larger of the two numbers

6. A streaming service says: 'A subscription includes 5 free movie rentals right when you sign up, and after that, each additional rental costs $3.' Someone reading quickly says: 'so $3 is the amount you start with, since it's the first number in the pricing part of the sentence.' What is wrong with this reasoning, and what should the starting amount and constant change actually be?

(A) The reasoning is wrong because there is no starting amount in this situation at all, only a constant change of $3 per rental

(B) The reasoning is correct: $3 is the starting amount because it appears in the pricing sentence, and 5 is the constant change

(C) The reasoning wrongly picks a number by its position in the sentence rather than by what it means; the starting amount is really 5 free rentals, and the constant change is $3 per rental

(D) The reasoning is wrong because $3 should instead be treated as 3 rentals, since the letter or number in a cost expression always refers to a count of objects

7. A farmer's storage silo starts a season holding 900 bushels of grain, and the farmer removes 25 bushels every week to sell at market. A student writes the constant change as '25w' and says, 'that means 25 weeks.' What is the error, and what does 25w actually represent?

(A) The error is that the constant change should be written as w/25 instead of 25w, since rates are always written as one quantity divided by another

(B) The student is treating the letter w as if it labels weeks themselves, rather than as a variable for the number of weeks; 25w actually represents the total bushels removed after w weeks

(C) There is no error; 25w correctly means 25 weeks

(D) The error is that the starting amount should be included inside 25w, since all quantities in the situation must appear in a single term

MATH · UNIT 1 · LESSON 2

Computing slope from two points



Goal: The student computes slope from two points or a table using (change in y)/(change in x) and correctly labels the units of the result in context.

About 45 minutes

Teach it FOR THE PARENT

1. **Say:** Two ladders lean against a wall. One is steep, one is more spread out. If I told you 'this ladder rises 8 feet for every 2 feet it moves out from the wall,' could you picture which one that is? That number, 8 feet up for every 2 feet across, is exactly what we're computing today. It's called slope.
   Look for: Student can say slope compares vertical change to horizontal change, even informally, based on the graphing they did in 8th grade.

2. **Say:** Here's the rule: slope equals (change in y) divided by (change in x). If you have two points, subtract the y-values to get change in y, subtract the x-values in that same order to get change in x, then divide. The answer always needs units, like miles per hour or dollars per hour, because slope is a rate, it's telling you how fast one thing changes compared to the other.
   Look for: Student can restate the formula m = (y2-y1)/(x2-x1) and say why units matter (it's a rate).

3. **Say:** Here's the car problem worked out step by step: at 2 hours, 80 miles; at 5 hours, 200 miles. Step 1: change in y is 200 - 80 = 120 miles. Quick check, did I subtract in the same order for x? Step 2: change in x is 5 - 2 = 3 hours. Step 3: divide, 120 ÷ 3 = 40 miles per hour. Why miles per hour, not just 40? Because I divided a mile-quantity by an hour-quantity.
   Look for: Student follows each subtraction step and can explain why the units are 'miles per hour' rather than a bare number.

4. **Say:** Your turn, fresh numbers. A tank's water level: at 3 minutes it's at 12 inches, at 8 minutes it's at 32 inches. Find the slope and give its units. Show your change in y, your change in x, then divide.
   Look for: Change in y = 32-12=20 inches; change in x = 8-3=5 minutes; slope = 4 inches per minute, with units stated.

5. **Say:** You've now got the routine down: subtract y's, subtract x's in the same order, divide, and always name the units. The practice set ahead uses points and tables, same three steps every time. Watch for the ones that try to trick you into skipping the units or reversing the subtraction order.
   Look for: Student is ready to move into independent practice items.

If they get stuck

- *The equals sign means 'now write down the answer' rather than 'these two things are the same value', so an equation like 3 + 4 = 7 feels fine but 7 = 3 + 4 looks 'backwards', and an equation with expressions on both sides (used to find where two plans cost the same) is confusing because there's nothing to 'work out' on one side.*
  In the equation-writing and intersection lessons, deliberately present equations in reflexive and operations-on-both-sides form from the first worked example (not just a-op-b=c), and have the student verbally restate what each side represents in the context before manipulating symbols.
- *In an expression like '3b', the letter b is a label for the object itself ('3 buns') rather than a variable standing for a number (the number of buns), so combining or comparing terms with letters gets treated like combining objects rather than combining quantities.*
  In the equation-writing and function-notation lessons, require the student to state the units of every variable and every term aloud or in writing before symbols are manipulated ('m is a number of months, so 15m is a number of DOLLARS, not months'), do this as a fixed step, not a one-time warning.
- *When translating a comparison sentence like 'there are six times as many minutes used by plan A as by plan B' into symbols, the student writes the equation in the order the words appear (6A = B) rather than matching quantities correctly (A = 6B).*
  Teach a check-with-a-number step as a fixed routine: pick a small value for one quantity, compute what the other should be from the sentence, then test both candidate equations against that pair before committing.

Read together FOR THE STUDENT

Slope tells you how steep a line is and which way it tilts. Find two points on the line or in a table. See how much y goes up or down (that's 'rise'), then see how much x goes across (that's 'run'). Slope = rise/run, or (change in y)/(change in x). Example: if a plant grows from 4 inches to 10 inches while 3 weeks pass, slope = (10-4)/3 = 2 inches per week. Always attach the units, 'inches per week', because slope is a rate, not just a bare number.

Going further: Slope is the ratio (change in y)/(change in x) between two points, written m = (y2 - y1)/(x2 - x1). Pick two points, subtract y-values in the same order you subtract x-values, and divide. The result is a rate of change: it tells you how much the output (y) changes for every one unit the input (x) changes. In context problems, label the units of y over the units of x, such as dollars per hour or feet per second, because the number alone does not communicate the rate.

Watch it work

A car's distance from home is recorded: at 2 hours it has gone 80 miles; at 5 hours it has gone 200 miles. Find the slope and label its units.

1. Identify the two points as (x, y) pairs: (2 hours, 80 miles) and (5 hours, 200 miles), because slope needs an (x1,y1) and (x2,y2).
2. Compute change in y: 200 - 80 = 120 miles. This is the rise, the amount the distance changed.
3. Compute change in x, using the same point order as the y-subtraction: 5 - 2 = 3 hours. This is the run.
4. Divide: 120 miles ÷ 3 hours = 40 miles per hour. The units come from dividing the y-unit by the x-unit, so the answer is not just '40', it is 40 miles per hour, the car's speed.

The idea: Slope is a rate: always divide the y-unit by the x-unit to name what the number means.

A table shows the cost of renting a bike for different numbers of hours: 1 hour costs $8, 3 hours costs $16, 5 hours costs $24. Find the slope between the first and last rows and label its units.

1. Choose two rows to use as points: (1 hour, $8) and (5 hours, $24). Any two rows work for a line with constant slope, but you must keep the same two rows for both subtractions.
2. Compute change in y (cost): 24 - 8 = 16 dollars.
3. Compute change in x (hours), same row order as above: 5 - 1 = 4 hours.
4. Divide: 16 dollars ÷ 4 hours = 4 dollars per hour. This means every extra hour of rental adds $4 to the cost.

The idea: When two points come from a table, you can pick any two rows, the ratio of change in y to change in x stays the same for a line, as long as you keep row order consistent in both subtractions.

Practice: Computing slope from two points

1. Find the slope between the points (1, 5) and (4, 17), and give the units if the point form is (hours, miles).

(A) 4 miles per hour (B) 12 miles per hour (C) 4 hours per mile

(D) 3 miles per hour

2. A table shows temperature over time: at 0 minutes it's 20°C, at 4 minutes it's 36°C. What is the slope, with units?

(A) 4 degrees Celsius per minute (B) 16 degrees Celsius per minute

(C) 0.25 minutes per degree Celsius (D) 4 minutes per degree Celsius

3. Use this table: hours: 2, 4, 6; cost: $10, $18, $26. Find the slope from the first row to the last row, with units.

(A) 4 hours per dollar (B) 16 dollars per hour (C) 4 dollars per hour

(D) 8 dollars per hour

4. Points (3, 40) and (7, 24) show (days, gallons of water in a leaking tank). Find the slope and its units.

(A) -4 gallons per day (B) 4 gallons per day (C) -4 days per gallon

(D) -16 gallons per day

5. A gym membership table shows: 1 month costs $45, 2 months costs $70, 4 months costs $120. Find the slope using the second and third rows (2 months, $70) and (4 months, $120), with units.

(A) 50 dollars per month (B) 37.5 dollars per month (C) 2 months per dollar

(D) 25 dollars per month

6. A table lists (weeks, plant height in cm): week 2 = 14 cm, week 5 = 14 cm, week 9 = 14 cm. Find the slope between week 2 and week 9, with units.

(A) 14 centimeters per week (B) 0 centimeters per week (C) 7 centimeters per week

(D) There is no slope because the values repeat

7. Points (x, y) = (6, 15) and (6, 42) come from a table where x is minutes and y is total pages read. A student says the slope is 27 pages per minute. What is wrong with this claim?

A. The student forgot to divide by 2 since there are two points in the pair, so the real slope is 13.5 pages per minute and not 27

B. The student should have written 27 minutes per page instead

C. 27 pages per minute is correct as written

D. Change in x is 6-6=0, and dividing by 0 is undefined, so this pair of points cannot give a slope at all

8. A cell phone plan table shows minutes used by two plans over time: at 2 months, Plan A has used 100 minutes and Plan B has used 100 minutes. A student is asked to find the slope of Plan A's minutes over time using points (2 months, 100 minutes) and (5 months, 250 minutes). They write: change in y = 5-2=3, change in x = 250-100=150, slope = 3/150. What error did they make?

A. 3/150 is correct as written, because it simplifies to the same slope in the end

B. They swapped which quantity is change in y and which is change in x

C. They forgot to add the units at the end

D. They should have used Plan B's data instead of Plan A's data to find the slope for this question

Slope is the same everywhere

Goal: The student explains why the slope computed between any two points on the same line is identical, using the idea of equal-interval accumulation of change rather than recomputation of the formula.

About 45 minutes



Teach it FOR THE PARENT

1. **Say:** Here's a weird claim: if I pick ANY two points on a straight line, right next to each other, or way far apart, and I compute the slope both times, I get the exact same number. That seems like it shouldn't be true. Why would far-apart points and close-together points give you the same answer?
   Look for: Student notices this is surprising and offers an initial guess, even a rough one, like 'because it's a straight line so it can't bend', this is a hunch we'll sharpen, not a final answer.

2. **Say:** Think of a staircase where every single step is the same height and same width. If you climb from step 2 to step 5, that's 3 steps, so you go up 3 step-heights. Climb from step 5 to step 9, that's 4 steps, so you go up 4 step-heights. The 'per-step' climb never changed. A line does this with tiny steps: for every 1 unit you move right, it always adds the SAME amount of y. That's the one number we call slope.
   Look for: Student can restate, in their own words, that a line adds the same y-amount per x-unit everywhere, not just between one specific pair of points.

3. **Say:** A line goes through (2,5) and (5,14). x changed by 3, y changed by 9, that's 3 units of y per unit of x. If I now go from (5,14) to (7?) on the SAME line, how much bigger should y get, and why, without me telling you the new point?
   Look for: Student reasons: x changes by 2, and since every unit of x adds 3 to y on this line, y should increase by $2 \times 3 = 6$, landing at 20, reasoning from the constant per-unit rate, not from a memorized rule.

4. **Say:** Now you try explaining it forward. A line passes through (0,1) and (4,13). Someone picks a totally different pair on the same line, far away: (10,31) and (12,37). Without recomputing both slopes and just comparing the numbers, explain WHY those two slopes have to match.
   Look for: A relational explanation: identifies the constant rate from the first pair (3 per unit), and argues the second pair is just more copies of the same repeating unit step, so it must show the same rate, not merely 'I calculated both and they're equal.'

5. **Say:** Quick gut-check: does 'the slope is the same everywhere on this line' mean the line is not curving anywhere? Say why in one sentence.
   Look for: Student connects constant per-unit rate to straightness, if the rate ever changed, the graph would bend, which is exactly what distinguishes lines from curves.

6. **Say:** You've now got the real reason slope stays the same across any two points on a line, it's one repeating rate, just measured over different-sized chunks. The practice set is going to ask you to explain this for new point pairs, and catch some explanations that are wrong. Read carefully, some answers just recompute numbers instead of explaining the invariance.
   Look for: Student moves into practice ready to distinguish 'explains why' answers from 'just recomputed and checked' answers.

If they get stuck

- *The equals sign means 'now write down the answer' rather than 'these two things are the same value', so an equation like 3 + 4 = 7 feels fine but 7 = 3 + 4 looks 'backwards', and an equation with expressions on both sides (used to find where two plans cost the same) is confusing because there's nothing to 'work out' on one side.*
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- *When translating a comparison sentence like 'there are six times as many minutes used by plan A as by plan B' into symbols, the student writes the equation in the order the words appear (6A = B) rather than matching quantities correctly (A = 6B).*
  Teach a check-with-a-number step as a fixed routine: pick a small value for one quantity, compute what the other should be from the sentence, then test both candidate equations against that pair before committing.

Read together FOR THE STUDENT

Picture walking up a staircase where every step is exactly the same height and width. If you measure your rise from step 2 to step 5, you climb 3 steps and go up 3 step-heights. If you measure from step 5 to step 9, you climb 4 steps and go up 4 step-heights. Different chunks, but each chunk climbs at the same rate, one step-height per step. A line works the same way: pick any two points, and the 'rise per run' you get is the same number, because the line adds the same amount of change for every equal chunk of horizontal distance.

Going further: Slope measures a rate of change: how much the output (y) changes for each unit the input (x) changes. On a straight line, this rate does not depend on which two points you pick, because a line is built from equal-sized vertical jumps stacked on equal-sized horizontal jumps, everywhere along it. So going from point (2,5) to point (2,5) to point (6,17) covers 4 horizontal units and 12 vertical units, 3 per unit, and going

from (6,17) to (9,26) covers 3 horizontal units and 9 vertical units, still 3 per unit, because each horizontal unit always carries the same vertical increment.

Watch it work

A line passes through (1, 4) and (4, 13). Another point on the same line is (6, 19). Explain why the slope from (1,4) to (4,13) must equal the slope from (4,13) to (6,19), without recomputing both and just checking they match.

1. From (1,4) to (4,13): x increases by 3 and y increases by 9. Think of this as 3 equal 'unit steps' in x, each one carrying the same amount of y-change, because that is what it means for the points to lie on one straight line, so each unit step carries $9 \div 3 = 3$ units of y.
2. From (4,13) to (6,19): x increases by 2. Because the line adds exactly 3 units of y for every 1 unit of x everywhere along it (not just between the first two points), these 2 steps must carry $2 \times 3 = 6$ units of y, and indeed $19 - 13 = 6$.
3. So the slope between the second pair isn't a coincidence that happens to equal 3, it has to equal 3, because both pairs are just different-sized chunks of the same constant per-unit rate. Chunk size changed (3 steps vs. 2 steps); the rate per step did not.
4. This is why slope is called 'the' slope of a line, not 'a' slope between two particular points, every pair of points is sampling the same underlying rate.

The idea: Slope between any two points on a line is not a fresh fact each time, it's the same constant per-unit rate, sampled over a different-sized chunk of the line.

A student says: 'Slope from (0,2) to (3,11) is 3, but I bet slope from (3,11) to (10,32) is different, since it's a totally different pair of points far away on the line.' Explain what's wrong with this reasoning, using the idea of equal-interval accumulation.

1. The student is treating each pair of points as its own separate situation, as if the line could 'change its mind' about how fast it climbs between one stretch and another. But a straight line is defined by adding the same y-amount for every same-sized x-amount, everywhere, that's what makes it straight rather than curved.
2. Check: from (0,2) to (3,11), x changes by 3 and y changes by 9, which is 3 units of y per unit of x. From (3,11) to (10,32), x changes by 7 and y changes by 21, also exactly 3 units of y per unit of x.
3. The '7 units of x' stretch is just 7 copies of the same single-unit step that made up the first stretch's 3 units of x. Since each single-unit step always adds 3 to y on this line, any stretch, no matter how long or where it sits, will show the same 3-per-1 rate when you divide total y-change by total x-change.
4. So distance along the line doesn't matter and location doesn't matter; what matters is that every stretch is built out of the same repeating unit step, which is why the ratio never changes.

The idea: Being 'far apart' on the line doesn't change the rate, because a longer stretch is just more copies of the same unit step, not a different kind of step.

 Name: _____

Practice: Slope is the same everywhere

1. A line passes through (1,3) and (4,12). Between these points, x changes by 3 and y changes by 9. Which statement correctly describes the per-unit rate on this line?

- **A** Every time x increases by 1, y increases by 9, since that's the total change given.
- **B** The rate only applies between x=1 and x=4; a new pair of points would need a new rate.
- **C** Every time x increases by 1, y increases by 3, and this holds everywhere on the line.
- **D** You can't know the per-unit rate without picking a third point to check.

2. A line passes through (0,4) and (5,19). Priya computes slope using (0,4) and (5,19) and gets 3. She then picks a new pair on the same line, (5,19) and (9,31), and gets 3 again. Which explanation shows she understands WHY the two slopes had to match, not just THAT they matched?

- **A** "Both pairs of points are on the same line, and the same line always gives the same numbers when you calculate anything from it, so it doesn't really matter which two points you pick or how far apart they are, the answer will always match."
- **B** "The x-values in both pairs are close to each other, so the slopes end up close in value."
- **C** "I calculated both of the slopes with the formula and they both came out to the same number, 3, so the claim is confirmed, and checking a third pair would only confirm it again because that is just how the formula works."
- **D** "Since the line adds 3 units of y for every 1 unit of x everywhere along it, the second stretch of 4 x-units must add $4 \times 3 = 12$ units of y, which is exactly what $31 - 19$ gives, so the match isn't a coincidence."

3. Which statement best explains why the slope between (2,7) and (8,25) on a line equals the slope between (8,25) and (11,34) on the same line?

(A) The slope only stays the same because 8 is included in both pairs; if no point were shared, the slopes could differ.

(B) Since 6 is exactly double 3, the y-change must exactly double too, and that's the only reason the ratios match.

(C) Both pairs are on the same line, and a longer x-distance (6 units vs 3 units) simply produces a proportionally larger y-distance built from the same constant rate per unit of x.

(D) Both pairs happen to use points where y is odd, which keeps the slope constant.

4. A line contains (3,10), (5,16), and (9, y). A student argues: 'Since (9,y) is much farther from (3,10) than (5,16) is, the slope between (3,10) and (9,y) will be a different, smaller number than the slope between (3,10) and (5,16).' What is wrong with this reasoning?

(A) The student is confusing distance along the line with the per-unit rate, the rate stays the same no matter how far apart the two points are, because farther points just mean more copies of the same unit step.

(B) The student's mistake is a simple computation error; if they redo the arithmetic carefully with the right value of y, they'll find that the two slopes actually do come out different, just as they expected, since the farther point covers more distance.

(C) The student is correct, because slope naturally decreases as x-values get larger.

(D) The student is right that farther-apart points give smaller slopes, but wrong about which point pair is farther apart.

5. Two students discuss why slope is constant on a line. Student 1 says: 'It's constant because whenever we plug two points into the slope formula, the algebra always simplifies to the same fraction, no matter which points we pick, that's just how the formula works.' Student 2 says: 'It's constant because the line is built by adding the same y-amount for every equal x-amount, everywhere, so any two points are just measuring different-sized samples of that one repeating rate.' Which student's explanation actually addresses WHY slope is invariant, rather than restating THAT it is?

(A) Student 2, because it explains the invariance using the idea that equal x-steps always carry equal y-amounts on a line.

(B) Both are equally good, since they reach the same conclusion.

(C) Neither of the two students, because slope invariance can only ever be shown by checking many numerical examples, not by any argument.

(D) Student 1, because the algebra explains the mechanism directly.

6. A line passes through (4,9) and (7,18). A student is asked to find the slope between (7,18) and (13, y) without being told y, and to explain their method. Which explanation demonstrates understanding of slope invariance, rather than just knowing a procedure to find y?

(A) "Since the line adds 3 units of y per unit of x everywhere, and (13,y) is 6 units of x past (7,18), y must be 18+6×3=36, the slope from (7,18) to (13,36) is then guaranteed to be 3 because I built it from that same constant rate."

(B) "Since 13 is bigger than 7, y will be bigger than 18, so I just need to pick any value greater than 18 for y and then the slope between the two points will come out positive, which is all that matters for a line that is going up, so any such value will work here."

(C) "I'll graph both of the points on the same grid and see if the line looks straight between them; if it does look straight, then the slope between (7,18) and (13,y) must be the same as before, and if it looks bent, then the slope must have changed to some new number."

(D) "I'll use the slope formula (y2-y1)/(x2-x1) with the new points once I know what y is, then simplify the fraction to lowest terms, and that will be the slope between (7,18) and (13,y), since the formula is the only reliable way to get a slope from two points."

7. A student was explicitly told the rule 'slope is the same between any two points on a line' and memorized it. On a quiz, asked to explain WHY this is true for a new line through (1,6) and (5,22), the student writes: 'It's true because that's the rule, slope is always the same on a line.' Why does this response NOT demonstrate the understanding this lesson is targeting?

(A) Because the rule itself is only true for some lines, not all of them.

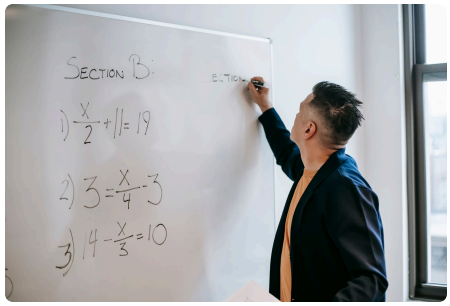
(B) Because it restates the rule as a fact to recall rather than explaining it using the idea that equal x-steps carry equal y-amounts.

(C) Because the student's rule is stated in the wrong mathematical vocabulary.

(D) Because the student should have used the specific numbers (1,6) and (5,22) in their answer no matter what.

MATH · UNIT 1 · LESSON 4

Which representation came first



Goal: Given a table, a graph, and a verbal description of three DIFFERENT linear situations, the student sorts them by which representation was generated FIRST and justifies the sort using surface cues that turn out to be unreliable, then revises the sort using structural cues (constant difference, common ratio check).

About 50 minutes

Teach it FOR THE PARENT

1. **Say:** Here are three things. A table: when x is 0, 1, 2, 3, then y is 5, 9, 13, 17. Imagine a graph: a line through the points (0, 2) and (4, 10), drawn with the y-axis marked in tiny steps of 0.5, so the line looks very steep on the page. And a story: 'A tank starts with 5 gallons and gains 4 gallons every minute.' All three might be hiding the exact same rule, or they might not. One of these three was made FIRST, and the other two were built to match it. Before I tell you how to check, take a guess: which one do you think came first, and why?
   Look for: Any guess based on a surface feature (looks steep, biggest numbers, most detailed story), this is expected and will be revisited, not corrected yet.

2. **Say:** Your eyes lie to you here. A graph can look steep just because of how the axis is scaled. A table can look 'more important' just because its numbers are bigger. None of that tells you the real rule. The only trustworthy test is: does the difference in y stay the same every time x goes up by the same amount? That's called a constant difference, and it's the real fingerprint of a linear rule.
   Look for: Nothing to check yet, this sets up the check move.

3. **Say:** In this table: x = 0, 1, 2, 3 gives y = 6, 10, 14, 18. Does this table have a constant difference? What is it?
   Look for: Student computes 10-6=4, 14-10=4, 18-14=4 and states the constant difference is 4, confirming it's linear with rate 4.

4. **Say:** Now let's sort the same three together: the table (y = 5, 9, 13, 17 when x = 0, 1, 2, 3), the line through (0, 2) and (4, 10), and the tank story that starts at 5 gallons and gains 4 gallons every minute. I won't trust which one looks fanciest. For the table, the difference is 4 every time, so its rule is y = 4x + 5. For the line, the slope is (10 minus 2) divided by (4 minus 0), which is 2, so its rule is y = 2x + 2, even though it looked steep. For the story, it starts at 5 and gains 4 each minute, so its rule is also y = 4x + 5. The table and the story match. The line is a different rule that only looked similar because of how it was drawn.
   Look for: Follow along; this models the full sorting process: checking table differences, checking the line's slope with two far-apart points, and extracting rate and start from the story.

5. **Say:** In the sorting we just did, the line through (0, 2) and (4, 10) looked steep, but its actual slope came out to 2, not 4. Why did it LOOK steeper than a slope-2 line usually looks?
   Look for: Student identifies that the y-axis was scaled in units of 0.5 rather than 1, which visually exaggerates steepness without changing the actual slope ratio.

6. **Say:** Your turn, a fresh set. Table X: x=0,3,6,9 → y=4,13,22,31. Graph Y: passes through (0,1) and (5,16), drawn with normal axis scaling. Story Z: 'A phone starts at 4% battery and charges 3% every minute.' Sort these three by their actual rule, do two of them match? Show your constant-difference or slope check for each, not just your final answer.
   Look for: Student computes Table X rate = (13-4)/3 = 3 (and confirms with another pair), Graph Y slope = (16-1)/5 = 3, Story Z rate = 3 starting at 4. Correctly identifies Table X's rule as y=3x+4, Graph Y's rule as y=3x+1, these do NOT match despite similar rates, because starting values differ (4 vs 1). Story Z matches Table X exactly (y=3x+4). Reasoning is structural, not based on appearance.

7. **Say:** You just proved to yourself that looks don't tell you the rule, only the numbers do. Now you'll do this same sorting-and-checking on new sets, and some of the wrong answers in the practice will tempt you back toward the surface-cue guesses. Catch yourself and go back to computing.
   Look for: Readiness to move to independent items; no new content introduced here.

If they get stuck

- *The equals sign means 'now write down the answer' rather than 'these two things are the same value', so an equation like 3 + 4 = 7 feels fine but 7 = 3 + 4 looks 'backwards', and an equation with expressions on both sides (used to find where two plans cost the same) is confusing because there's nothing to 'work out' on one side.*
  In the equation-writing and intersection lessons, deliberately present equations in reflexive and operations-on-both-sides form from the first worked example (not just a-op-b=c), and have the student verbally restate what each side represents in the context before manipulating symbols.
- *In an expression like '3b', the letter b is a label for the object itself ('3 buns') rather than a variable standing for a number (the number of buns), so combining or comparing terms with letters gets treated like combining objects rather than combining quantities.*
  In the equation-writing and function-notation lessons, require the student to state the units of every variable and every term aloud or in writing before symbols are manipulated ('m is a number of months, so 15m is a number of DOLLARS, not months'), do this as a fixed step, not a one-time warning.
- *When translating a comparison sentence like 'there are six times as many minutes used by plan A as by plan B' into symbols, the student writes the equation in the order the words appear (6A = B) rather than matching quantities correctly (A = 6B).*
  Teach a check-with-a-number step as a fixed routine: pick a small value for one quantity, compute what the other should be from the sentence, then test both candidate equations against that pair before committing.

Read together FOR THE STUDENT

You can show the same linear rule three ways: a table of numbers, a graph, or a story in words. Someone made ONE of these first, then built the other two from it. Here's the trap: a table with big jumps between numbers can LOOK 'more important' or 'first,' and a graph that starts high up can look like it came first too, but looks lie. The real test is math: check if the difference between y-values stays the same every time x goes up by 1. That constant difference is the actual fingerprint of which rule is behind all three.

Going further: A linear relationship can be represented as a table, a graph, or a verbal description, and all three can be generated from the same underlying rule y = mx + b. Surface cues, which representation 'looks' most detailed, which graph has the steepest visual angle, which table has the biggest numbers, do not reveal which representation came first, because any representation can be redrawn or rewritten from any other. The reliable structural cue is the constant difference: in a table, equal x-steps must produce equal y-changes (checked by computing slope between multiple pairs of points, not just one). A graph's slope is a ratio of rise to run, not the steepness you see on the page, which changes depending on axis scaling. Differentiating the situations means comparing all three representations against this one structural test, not against how they appear.

Watch it work

Three items are given. Item T: a table showing x = 0,1,2,3 with y = 5,9,13,17. Item G: a graph passing through (0,2) and (4,10), drawn with a steep-looking line because the y-axis is scaled in units of 0.5. Item S: a story, 'A tank starts with 5 gallons and gains 4 gallons every minute.' A student's first guess: 'The graph came first because it looks the most dramatic and steep.' Sort them by which single rule they all came from, and decide if 'looks steep' is a valid reason.

1. Check Item T's structure first, not its appearance: from x=0 to 1, y goes 5→9 (+4). From 1 to 2, 9→13 (+4). From 2 to 3, 13→17 (+4). The difference is constant at +4 for every step of 1 in x. This constant difference is the real fingerprint, not how big the numbers look.
2. Check Item G the same way, using the ratio, not the visual steepness: slope = (10-2)/(4-0) = 8/4 = 2. The axis scaling made the line look steep on the page, but the actual rate of change is 2, not 4. 'Looks steep' was a surface cue and it was wrong, the axis choice tricked the eye, not the math.
3. Check Item S: starts at 5, gains 4 per minute. That means rate = 4, start = 5. Matching this to a rule: y = 4x + 5.
4. Compare all three rules: Item T gives y = 4x + 5 (starts at 5 when x=0, rate 4). Item S also gives y = 4x + 5. Item G gives y = 2x + 2, a different rule entirely. So T and S came from the same original rule; G is a different linear situation that only looked similar because of how its graph was scaled.

The idea: Whether a representation 'looks' bigger, steeper, or more dramatic tells you nothing about its underlying rule, only a computed constant difference or ratio, checked across more than one pair of points, tells you which situations share the same source rule.

A student is given: Table A (x=0,2,4,6 → y=3,11,19,27), Table B (x=0,1,2,3 → y=3,7,11,15), and a claim: 'Table A must be linear because its y-values jump by 8, which is bigger and more consistent-looking than Table B's jumps of 4.' Decide whether 'bigger, more consistent-looking jumps' is a valid reason to call something more linear, and check both tables structurally.

1. Restate what 'linear' actually requires: equal y-changes for equal x-changes, not merely 'jumps that look consistent' or 'jumps that are large.' Size of the jump is irrelevant; what matters is whether the jump PER UNIT of x-change stays the same throughout the whole table.
2. Check Table A's x-steps: they go up by 2 each time (0,2,4,6), and y goes up by 8 each time. Rate = 8/2 = 4 per unit of x. This is constant across all three intervals, so Table A is linear with slope 4.
3. Check Table B's x-steps: they go up by 1 each time, and y goes up by 4 each time. Rate = 4/1 = 4 per unit of x. This is also constant, so Table B is linear with slope 4 too.
4. Compare the conclusion to the student's original reasoning: both tables represent the exact same rule (slope 4, since Table A's jump of 8 over 2 x-steps reduces to the same per-unit rate as Table B's jump of 4 over 1 x-step). The claim that Table A is 'more linear' because its jumps look bigger is false, jump size alone, without dividing by the x-step, tells you nothing.

The idea: A big jump in y means nothing on its own, you must divide by the matching jump in x before comparing two tables, because unequal x-steps make raw jump sizes impossible to compare directly.

 Name: _____

Practice: Which representation came first

1. A table shows x = 0, 1, 2, 3 with y = 2, 6, 10, 14. What is the constant difference in y for each step of 1 in x?

(A) It changes each time, so there is no constant difference. (B) 14 (C) 4 (D) 2

2. A graph passes through (0, 3) and (2, 11). What is the slope, computed as a ratio (not by how the line looks on the page)?

(A) 8 (B) 2

(C) It depends on how the graph is scaled, so it can't be found from these two points alone. (D) 4

3. A story says: 'A candle starts at 10 inches tall and burns down 2 inches every hour.' Which equation matches, using h for hours passed and y for candle height?

(A) y = 10 - 2h (B) y = 2 - 10h (C) y = 10 + 2h (D) y = 12h

4. Table P: x = 0, 1, 2, 3 → y = 5, 12, 19, 26. Table Q: x = 0, 2, 4, 6 → y = 5, 19, 33, 47. A student says, 'Table Q must have a bigger rate because its y-values jump by 14 while Table P only jumps by 7.' Is the student right?

(A) It can't be determined without a graph.

(B) Yes, Table Q's jump of 14 is bigger than Table P's jump of 7, so Table Q has the bigger rate.

(C) No, once you divide each jump by its matching x-step, both tables have the same rate of 7.

(D) No, Table P has the bigger rate because its numbers are smaller and easier to work with.

5. A graph is drawn with the y-axis marked in units of 0.25 (so grid lines are close together), passing through (0, 4) and (4, 8). Another graph is drawn with the y-axis marked in normal units of 1, passing through (0, 4) and (4, 8) as well. A student says the first graph 'came from a steeper, faster-changing situation' because it looks steeper on the page. Evaluate this claim.

(A) It can't be evaluated one way or the other without knowing what units the x-axis uses on each of the two graphs, since the slope depends on those units as well.

(B) The claim is true, but only because the first graph has more grid lines drawn on it, which means that more data points were used to draw the line, and more data points always make the rate of change larger.

(C) The claim is false, both graphs pass through the same two points, so both have the identical slope of 1, regardless of how the axis is scaled.

(D) The claim is true: the first graph's tighter axis spacing means that the underlying rate of change is actually faster, since the line has to cross many more grid lines to get from one point to the other.

6. A story says: 'There are five times as many minutes on Plan A as on Plan B.' A student writes the equation $5A = B$ to match this sentence. A friend checks it by picking real numbers: if Plan B has 20 minutes, Plan A should have 5 times that, or 100 minutes. Does $5A = B$ give the right relationship?

(A) No, the equation should be $B = 5A$, since B is mentioned second in the sentence.

(B) Yes, $5A = B$ correctly matches the word order of the sentence, since '5 times as many' comes right before 'Plan A' isn't quite how it's read, but the 5 stays with A regardless of order.

(C) It doesn't matter which way it's written, since both equations describe the same relationship between A and B.

(D) No, plugging in $B = 20$ gives $5A = 20$, so $A = 4$, but Plan A should be 100. The equation should be $A = 5B$ instead.

7. Three items describe possibly-related linear situations. Table M: x=0,1,2,3 → y=8,11,14,17. Graph N: passes through (0,8) and (3,17), axis scaled normally. Story L: 'A savings account starts at 11 dollars and grows by 3 dollars every week.' Exactly two of these three share the same underlying rule. Which two?

(A) All three of the items share exactly the same rule, since all three of them have a rate of 3, and the rate is the only thing that decides whether two linear rules are the same.

(B) Graph N and Story L: both of them describe growth over time, so they must be the matching pair; Table M is the odd one out because a table is not about time.

(C) Table M and Graph N, both have rate 3 and start 8, giving y = 3x + 8; Story L has a different start value (11), so it's the odd one out.

(D) Table M and Story L: both of them mention the number 3 as a key value, so they must share the same rule; Graph N is the odd one out because it never mentions 3.

Writing equations in slope-intercept form



Goal: The student writes a linear equation in slope-intercept form from a context, a table, or two points, choosing the correct method for the given starting information.

About 50 minutes

Teach it FOR THE PARENT

1. **Say:** Here's the thing about linear equations: nobody ever hands you the same starting information twice. Sometimes you get a story, sometimes a table, sometimes just two dots on a graph paper. Today's whole skill is NOT computing, you already know how to find slope and solve one-step equations. Today's skill is: look at what you're given, and decide which move to make first.
   Look for: Nothing to assess yet, just attention on the idea that the input format drives the method.
2. **Say:** Every linear rule is y = mx + b. b is where you start (x = 0), m is how much y changes each time x goes up by 1. If your problem is a STORY, read m and b straight out of the words, no math needed yet. If it's a TABLE or TWO POINTS, you have to compute m first using two rows, then plug one point in to solve for b.
   Look for: Student can restate: story = read directly, table/points = compute m first, then solve for b.
3. **Say:** Let's work through the gym membership problem. I ask myself first: is this a story, a table, or points? It's a story, words. So I hunt for the fixed starting amount and the per-month rate, no calculating. Check: where's the $40 sitting in my equation, and where's the $15?
   Look for: Student points to $40 as b (present at t=0) and $15 as m (rate per month), not reversed.
4. **Say:** Now the table one. First question again: story, table, or points? Table. That means no rate is stated, I have to compute m from two rows: (20-11)/(5-2). Check: what do I get, and why did I subtract y's on top and x's on bottom, not the other way?
   Look for: Student computes m = 3 and can explain change in y over change in x, matched row to row correctly.
5. **Say:** Your turn, fresh problem: two points, (1, 4) and (4, 13). First tell me, story, table, or points? Then find m, then use one point to solve for b, then write the full equation.
   Look for: Student names 'points', computes m = (13-4)/(4-1) = 3, substitutes to get b = 1, writes y = 3x + 1.
6. **Say:** You've now handled all three starting formats. In the practice set, I'm not going to tell you which method to use, that's the actual skill being tested. Read the problem, decide what you've got, then go.
   Look for: Student moves into practice expecting to self-identify format before computing.

If they get stuck

- *The equals sign means 'now write down the answer' rather than 'these two things are the same value', so an equation like 3 + 4 = 7 feels fine but 7 = 3 + 4 looks 'backwards', and an equation with expressions on both sides (used to find where two plans cost the same) is confusing because there's nothing to 'work out' on one side.*
  In the equation-writing and intersection lessons, deliberately present equations in reflexive and operations-on-both-sides form from the first worked example (not just a-op-b=c), and have the student verbally restate what each side represents in the context before manipulating symbols.
- *In an expression like '3b', the letter b is a label for the object itself ('3 buns') rather than a variable standing for a number (the number of buns), so combining or comparing terms with letters gets treated like combining objects rather than combining quantities.*
  In the equation-writing and function-notation lessons, require the student to state the units of every variable and every term aloud or in writing before symbols are manipulated ('m is a number of months, so 15m is a number of DOLLARS, not months'), do this as a fixed step, not a one-time warning.
- *When translating a comparison sentence like 'there are six times as many minutes used by plan A as by plan B' into symbols, the student writes the equation in the order the words appear (6A = B) rather than matching quantities correctly (A = 6B).*
  Teach a check-with-a-number step as a fixed routine: pick a small value for one quantity, compute what the other should be from the sentence, then test both candidate equations against that pair before committing.

Read together FOR THE STUDENT

A line has two jobs: a starting number (where it begins, called b) and a step number (how much it changes each time, called m). The rule looks like y = mx + b. Sometimes you're told the story straight out ('starts at $40, goes up $15 a month'). Sometimes you get a table of numbers and have to find the step yourself. Sometimes you just get two dots on a graph. Your first job, every time, is to figure out which of these three you were handed, THEN pick the matching method.

Going further: Slope-intercept form is y = mx + b, where m is the slope (rate of change) and b is the y-intercept (starting value). The method you use depends on what you're given: from a context, read m and b directly from the words. From a table, compute m = (change in y)/(change in x) using two rows, then substitute one point to solve for b. From two points, compute m the same way, then substitute either point to solve for b. The skill isn't any single method, it's correctly identifying your starting format before choosing one.

Watch it work

A gym membership costs $40 to join, then $15 per month. Write an equation for total cost C in terms of number of months t.

1. Identify the format: this is a context (words), not a table or points, so look for the starting value and the rate directly in the sentence, no computing needed.
2. Find b: the $40 join fee happens at t = 0 months, before any monthly charge, so b = 40.
3. Find m: '$15 per month' is the amount added for each additional month, so m = 15. Note that 15t means '15 dollars, t times', t counts months, it does not relabel the 15 as months.
4. Write the equation: C = 15t + 40.

The idea: In a context, m and b are usually stated as separate phrases, a fixed amount (b) and a rate per unit (m), so this method needs no computation, only correct reading.

A table shows: x = 2, y = 11; x = 5, y = 20; x = 8, y = 29. Write the equation in slope-intercept form.

1. Identify the format: this is a table of x-y pairs with no rate stated in words, so slope must be computed from two rows before anything else.
2. Compute m using two rows: m = (20 - 11)/(5 - 2) = 9/3 = 3.
3. Substitute one point, say (2, 11), into y = mx + b: 11 = 3(2) + b, so 11 = 6 + b.
4. Solve for b using one-step equation skills from Grade 8: subtract 6 from both sides, b = 5. Write the equation: y = 3x + 5.

The idea: From a table, slope is never given, it must be computed first from two rows, and only then can you solve for b the same way you would from two points.

 Name: _____

Practice: Writing equations in slope-intercept form

1. A plumber charges a $60 flat fee plus $25 per hour of work. Write an equation for total cost C in terms of hours h.

(A) C = 25 + 60 + h (B) C = 60h + 25 (C) C = 25h + 60 (D) h = 25C + 60

2. A table shows: x = 0, y = 5; x = 3, y = 14; x = 6, y = 23. Write the equation in slope-intercept form.

(A) y = 3x + 9 (B) y = 5x + 3 (C) y = 3x + 5 (D) y = 9x + 5

3. A line passes through the points (2, 7) and (5, 16). Write its equation in slope-intercept form.

(A) y = 3x + 1 (B) y = 9x + 1 (C) y = 3x + 7 (D) y = 3x - 1

4. A tank starts with 8 gallons of water and drains at 2 gallons per minute. Let m stand for the number of minutes. Which equation correctly represents the amount of water W left in the tank?

(A) 8 = -2m + W (B) W = -2 + 8m (C) W = -2m + 8 (D) W = 2m + 8

5. In a video game, a player's score is 4 times as many points as their opponent's score, plus a 50-point bonus. Let x = opponent's score and y = player's score. Which equation matches this correctly?

(A) x = 4y + 50 (B) y = 50x + 4 (C) y = 4x + 50 (D) 4y = x + 50

6. A table shows: x = 1, y = 20; x = 4, y = 11; x = 7, y = 2. Write the equation in slope-intercept form.

(A) y = -3x + 23 (B) y = 3x + 23 (C) y = -3x + 17 (D) y = -9x + 29

7. A moving company charges a flat fee plus a per-mile rate. A customer who moved 50 miles paid $350. A customer who moved 120 miles paid $560. Write the equation for total cost C in terms of miles d.

(A) C = 3d + 350 (B) C = 7d (C) C = 210d + 70 (D) C = 3d + 200

8. A candle is 15 cm tall and burns down at a steady rate. After burning for 4 hours, it is 11 cm tall. Write an equation for height H in terms of hours burned t.

(A) H = -1t + 15 (B) H = 1t + 15 (C) H = -1t + 11 (D) H = -4t + 15

Function notation and what it means

Goal: The student evaluates and interprets f(x) notation, stating what f(3) = 11 means in terms of the original context's two quantities.

About 45 minutes



Teach it FOR THE PARENT

1. **Say:** Quick question before we start: if I told you 'f(3) = 11' and nothing else, could you tell me anything real about the world? No, because f(x) is just a name for a rule, and right now it has no story attached. Today we attach the story back on.
   Look for: Student recognizes f(x) alone is abstract and needs context to mean something.
2. **Say:** f(x) means: put the number x into the rule, and f(x) is what comes out. If f(x) = 40 + 15x is a plumber's cost for x hours, then f(x) is dollars and x is hours, two different things. f(3) = 11 is really saying 'when the input is 3, the output is 11', you just have to say what those two numbers count.
   Look for: Student can restate that x and f(x) stand for two different quantities from the context, not the same thing.
3. **Say:** In g(m) = 5 + 3m, where m is minutes and g(m) is gallons, what does g(m) itself measure, gallons or minutes?
   Look for: Student answers 'gallons', the output is the tank amount, not the time.
4. **Say:** Watch me work through: a moving company charges f(h) = 60 + 25h dollars for h hours. What does f(4) = 160 mean? Step 1: h counts hours. Step 2: f(h) counts dollars. Step 3: match 4 to hours, 160 to dollars. Step 4: say it as one sentence, 4 hours of work costs $160. I can check it: 60 + 25(4) = 160. True.
   Look for: Student follows each step and can point to which number is the input and which is the output before I say the sentence.
5. **Say:** In that same problem, if someone said f(4) = 160 means '4 dollars buys 160 hours,' what did they get backwards?
   Look for: Student identifies they swapped which number counts hours and which counts dollars, input and output roles flipped.
6. **Say:** Your turn, fresh problem: a tank has g(m) = 5 + 3m gallons after m minutes. What does g(6) = 23 mean? Write it as one sentence naming minutes and gallons, then check it's true by plugging m = 6 into the rule.
   Look for: Student writes something like 'after 6 minutes, the tank has 23 gallons' and verifies 5 + 3(6) = 23.
7. **Say:** That's the whole move: name what x counts, name what f(x) counts, match the numbers, say the sentence. Every question coming up asks you to do exactly this for a new rule, same steps every time.
   Look for: Student is ready to apply the fixed four-step routine to new f(x) statements in the practice set.

If they get stuck

- *The equals sign means 'now write down the answer' rather than 'these two things are the same value', so an equation like 3 + 4 = 7 feels fine but 7 = 3 + 4 looks 'backwards', and an equation with expressions on both sides (used to find where two plans cost the same) is confusing because there's nothing to 'work out' on one side.*
  In the equation-writing and intersection lessons, deliberately present equations in reflexive and operations-on-both-sides form from the first worked example (not just a-op-b=c), and have the student verbally restate what each side represents in the context before manipulating symbols.
- *In an expression like '3b', the letter b is a label for the object itself ('3 buns') rather than a variable standing for a number (the number of buns), so combining or comparing terms with letters gets treated like combining objects rather than combining quantities.*
  In the equation-writing and function-notation lessons, require the student to state the units of every variable and every term aloud or in writing before symbols are manipulated ('m is a number of months, so 15m is a number of DOLLARS, not months'), do this as a fixed step, not a one-time warning.
- *When translating a comparison sentence like 'there are six times as many minutes used by plan A as by plan B' into symbols, the student writes the equation in the order the words appear (6A = B) rather than matching quantities correctly (A = 6B).*
  Teach a check-with-a-number step as a fixed routine: pick a small value for one quantity, compute what the other should be from the sentence, then test both candidate equations against that pair before committing.

Read together FOR THE STUDENT

f(x) is just a name for a rule that turns one number into another, like a machine. You drop a number in (that's x), and one number comes out (that's f(x)). If a plumber charges f(x) = 40 + 15x dollars for x hours of work, then f(3) = 85 means: when x is 3 hours, the cost is 85 dollars. The letter x is not a label for 'hours' the object, it's a stand-in for how many hours, a count. Read f(3) = 85 like a sentence about two real things: hours and dollars.

Going further: Function notation f(x) names the output that a rule produces for a given input x. Writing f(3) = 11 is a statement that when the input variable equals 3, the rule's output equals 11, it is one input-output pair from a relationship you already built as an equation this unit. To interpret it, you name what x counts and what f(x) counts in the original context, then say the two matched values in a sentence: 'when [x's meaning] is 3, [f(x)'s meaning] is 11.' The variable x is a quantity, not a label for the object itself.

Watch it work

A moving company charges a flat fee plus an hourly rate. The cost in dollars for h hours of work is f(h) = 60 + 25h. What does f(4) = 160 mean?

1. Identify what the input variable counts: h counts the number of hours of work, this comes straight from how the equation was built earlier in the unit, so nothing new is being defined here.
2. Identify what the output f(h) counts: f(h) is the total cost in dollars, the thing the equation was built to calculate.
3. Match the numbers to their roles: in f(4) = 160, the input is 4, so h = 4 hours; the output is 160, so f(h) = 160 dollars.
4. State the sentence connecting both quantities: f(4) = 160 means that 4 hours of work costs 160 dollars in total, plug h = 4 into 60 + 25h to check: 60 + 25(4) = 60 + 100 = 160, confirming the pair is correct.

The idea: f(3) = 11 is never just 'plug in 3, get 11', it's a paired statement about the two real quantities the function connects, and both numbers need their unit and role named.

A tank starts with 5 gallons of water and is filled at a constant rate. The number of gallons after m minutes is g(m) = 5 + 3m. What does g(6) = 23 mean, and is the statement true?

1. Name the two quantities the function connects: m is the number of minutes that have passed since filling started; g(m) is the number of gallons of water in the tank at that time.
2. Read the input and output out of the notation: in g(6) = 23, the input 6 stands for m = 6 minutes, and the output 23 stands for g(m) = 23 gallons.
3. Write the context sentence before checking anything: g(6) = 23 claims that after 6 minutes, the tank holds 23 gallons.
4. Check the claim against the rule, because a statement written in this notation can be false: g(6) = 5 + 3(6) = 5 + 18 = 23. The computed value matches the claimed output, so the statement is true.

The idea: An f(x) = value statement is a claim you can check against the original rule, interpreting it means saying what it claims about the two quantities, then verifying the claim if asked.

 Name: _____

Practice: Function notation and what it means

1. A parking garage charges f(h) = 5 + 2h dollars for h hours parked. What does f(3) = 11 mean?

(A) The garage charges 3 dollars per hour and made 11 dollars.

(B) 3 hours plus 11 hours equals the total parking time.

(C) Parking for 3 hours costs 11 dollars. (D) Parking for 11 hours costs 3 dollars.

2. A phone plan costs f(t) = 20 + 0.10t dollars for t text messages sent. What does f(50) = 25 mean?

(A) Sending 50 texts costs 25 dollars total.

(B) The plan has 50 dollars of base cost plus 25 texts.

(C) Sending 25 texts costs 50 dollars total. (D) Each text costs 50 cents, totaling 25 dollars.

3. A candle's height in inches after burning for m minutes is h(m) = 8 - 0.05m. Which sentence correctly interprets h(20) = 7?

(A) The candle loses 7 inches every 20 minutes.

(B) The candle burns 20 inches in 7 minutes.

(C) After 20 minutes of burning, the candle is 7 inches tall.

(D) After 7 minutes of burning, the candle is 20 inches tall.

4. A gym membership costs f(w) = 15w + 30 dollars, where w is the number of weeks. A student says: 'f(4) = 90 means the gym made 4 dollars from 90 weeks of membership.' What is wrong with this interpretation?

(A) It should say that the gym made 15 dollars from 30 weeks of membership, since those are the two numbers in the rule itself.

(B) It swaps the input and output, 4 is the number of weeks and 90 is the total cost in dollars, not the reverse.

(C) Nothing is wrong; both numbers describe money.

(D) It should say 4 weeks costs 15 dollars.

5. A rideshare fare in dollars for d miles is f(d) = 3 + 1.5d. Which of these is a correctly checked interpretation of f(6) = 12?

(A) 6 miles costs 12 dollars, and this checks out since 3 + 1.5(6) = 12.

(B) 12 miles costs 6 dollars, and this checks out since 3 + 1.5(12) = 21, not 6.

(C) 6 miles costs 12 dollars because that's what the numbers say.

(D) 6 miles costs 12 dollars, and this checks out since 3 + 1.5(12) = 6.

6. A water bill in dollars for u units used is f(u) = 12 + 0.08u. A classmate claims f(50) = 15. Which response correctly evaluates this claim?

(A) The claim is true; f(15) = 12 + 0.08(15) = 13.2, which rounds to match.

(B) The claim is false; f(50) = 12 + 0.08(50) = 16, not 15.

(C) The claim is true because 50 units and 15 dollars are both possible numbers for a water bill.

(D) The claim can't be checked without knowing the total number of units in a full month.

7. Two plans for a streaming service: Plan A costs A(m) = 8 + 2m dollars for m months, Plan B costs B(m) = 5 + 3m dollars for m months. A student is told B(3) = 14 and asked to interpret it. Which interpretation is correct AND uses the right variable roles?

(A) For Plan B, after 14 months, the total cost is 3 dollars.

(B) Plan A and Plan B cost the same, 14 dollars, after 3 months.

(C) For Plan B, 3 dollars buys 14 months of service.

(D) For Plan B, after 3 months, the total cost is 14 dollars.

8. A furniture rental company charges f(n) = 25n + 40 dollars for n months of rental. A customer says: 'f(n) = 40 means it costs 40 dollars no matter how many months, n.' What is the error in this statement, if any?

(A) The error is that f(n) should be renamed f(40) for this to make sense.

(B) The error is that n should be replaced with 25 in the statement, since 25 is the coefficient of n in the rule, and the coefficient is always the number that the input turns into once you substitute; so f(n) = 40 should really be written as f(25) = 40 and read as 25 months costing 40 dollars.

(C) There's no error at all in the customer's statement; 40 dollars is always the base fee regardless of how many months are rented, so f(n) = 40 is simply the correct way to say that the base fee stays fixed at 40 while n is allowed to be any number of months at all without changing the result.

(D) f(n) = 40 is not a statement about a specific number of months at all, it treats n as if it disappears, but n is still an unknown input, not a fixed value like the numbers we've interpreted.

Domain and range in context



Goal: Given a context with a natural restriction (e.g., number of weeks cannot be negative, or a tank empties and cannot go below zero), the student determines a realistic domain and range and explains what part of the algebraic line is NOT part of the model.

About 50 minutes

Teach it FOR THE PARENT

1. **Say:** Picture a bathtub draining. The equation for gallons left says y = 60 - 4x, where x is minutes. Plug in x = 20. You get y = -20. Can a tub have -20 gallons in it? What actually happens at minute 15, when the tub reaches zero?
   Look for: Student says a negative amount of water is impossible, and recognizes the tub just stays empty, the equation keeps going but reality stops.

2. **Say:** A line on paper never stops. But a real situation almost always does. Domain is the set of x-values that make sense in the story. Range is the set of y-values that actually happen. To find them, ask two questions: what values can x really take here, and does y hit a floor or ceiling because of the situation?
   Look for: Nothing to collect yet, this sets up vocabulary the student will use in the check.

3. **Say:** In the draining tub example, x is minutes and y is gallons. Which of these is a real restriction from the story: 'x can't be negative' or 'x can't be an even number'?
   Look for: Student picks 'x can't be negative' and can say why, you can't have negative minutes, but there's no reason minutes must be odd or even.

4. **Say:** Let's work the tank problem step by step: 80 gallons, draining at 5 gallons a minute, y = 80 - 5x. First: what's the smallest x can be, and why? Second: at what minute does the tank hit empty? Third: what does that tell us about the biggest x can be?
   Look for: $x \geq 0$ because minutes can't be negative; solving $80 - 5x = 0$ gives $x = 16$; so x stops making sense past 16 because gallons can't go negative.

5. **Say:** Now finish it: what's the range, the actual gallon amounts that occur, and which part of the algebraic line is NOT part of the real model?
   Look for: Range is $0 \leq y \leq 80$; the excluded parts are $x < 0$ and $x > 16$ (equivalently, everywhere the line dips below $y = 0$ or sits left of $x = 0$).

6. **Say:** Your turn, fresh problem: a parking garage charges y = 5 + 2x dollars, where x is hours parked. There's no fee cap mentioned, and hours can't be negative. Find the realistic domain and range, and say what part of the line is excluded.
   Look for: Domain $x \geq 0$, range $y \geq 5$, excluded part is $x < 0$ (which would also give $y < 5$), no upper bound exists here because the story never caps hours.

7. **Say:** Every question coming up gives you a story and an equation. Your job: figure out what real-world rule blocks part of the line, a can't-go-negative rule, a capacity ceiling, or both, and say exactly which piece of the line falls outside the model.
   Look for: Student is ready to apply the two-question method (check x's lower/upper limits, check y's floor/ceiling) to new contexts.

If they get stuck

- *The equals sign means 'now write down the answer' rather than 'these two things are the same value', so an equation like 3 + 4 = 7 feels fine but 7 = 3 + 4 looks 'backwards', and an equation with expressions on both sides (used to find where two plans cost the same) is confusing because there's nothing to 'work out' on one side.*
  In the equation-writing and intersection lessons, deliberately present equations in reflexive and operations-on-both-sides form from the first worked example (not just a-op-b=c), and have the student verbally restate what each side represents in the context before manipulating symbols.
- *In an expression like '3b', the letter b is a label for the object itself ('3 buns') rather than a variable standing for a number (the number of buns), so combining or comparing terms with letters gets treated like combining objects rather than combining quantities.*
  In the equation-writing and function-notation lessons, require the student to state the units of every variable and every term aloud or in writing before symbols are manipulated ('m is a number of months, so 15m is a number of DOLLARS, not months'), do this as a fixed step, not a one-time warning.
- *When translating a comparison sentence like 'there are six times as many minutes used by plan A as by plan B' into symbols, the student writes the equation in the order the words appear (6A = B) rather than matching quantities correctly (A = 6B).*
  Teach a check-with-a-number step as a fixed routine: pick a small value for one quantity, compute what the other should be from the sentence, then test both candidate equations against that pair before committing.

Read together FOR THE STUDENT

A line on a graph can go on forever in both directions. But real stories usually can't. If you're filling a tank with water, time can't be negative, you can't go back to 'before' filling started. And once the tank is full, it stays full even though the line keeps climbing on paper. So the real answer is just the piece of the line that matches the story. Everything else is math that doesn't happen.

Going further: Domain is the set of x-values (inputs) that make sense in the context. Range is the set of y-values (outputs) that actually occur. A linear equation itself has no restrictions, x and y can be any real number. But once you attach a context (weeks passing, gallons in a tank), the situation blocks off parts of the line: negative time, amounts below zero, amounts above a container's capacity. You state the restriction, then say which part of the algebraic line is excluded because of it.

Watch it work

A water tank holds 80 gallons. It starts full and drains at 5 gallons per minute. The equation is y = 80 - 5x, where x is minutes since draining started and y is gallons left. What is the realistic domain and range, and what part of the line is NOT part of the model?

1. Identify what x and y count: x = minutes passed, y = gallons in the tank. Minutes can't be negative, so $x \geq 0$ is required, this rules out the part of the line to the left of x = 0.
2. Find when the tank hits empty, because gallons can't go below 0: set y = 0, so $0 = 80 - 5x$, giving x = 16. The tank is empty at x = 16 and stays empty after that (it doesn't go negative), so the model only applies for $0 \leq x \leq 16$.
3. State the range from those x-values: at x = 0, y = 80 (full tank); at x = 16, y = 0 (empty). Since y decreases the whole time, the range is $0 \leq y \leq 80$.
4. Name what's excluded: the algebraic line also has points where $x < 0$ (negative minutes) and points where $x > 16$ (which give negative y, meaning negative gallons). Neither can happen in real life, so those two rays of the line are not part of the model, only the segment between (0, 80) and (16, 0) is.

The idea: A capacity limit on y (can't go below 0 gallons) creates a matching stopping point on x, the two restrictions are linked, not separate.

A gym membership costs a $30 sign-up fee plus $20 per month. The equation is y = 30 + 20x, where x is the number of months and y is total cost paid. What is the realistic domain and range, and what part of the line is NOT part of the model?

1. Identify what x and y count: x = number of months since joining, y = total dollars paid. Months can't be negative, you can't have joined -2 months ago, so $x \geq 0$.
2. Check whether x has an upper limit from the story: nothing in this context stops the membership from continuing indefinitely, so there is no upper bound stated, x can be any whole number 0, 1, 2, 3... (realistically whole months, not fractions of a month, unless the gym prorates).
3. Find the range by plugging in the smallest allowed x: at x = 0, y = 30 (just the sign-up fee, no months yet). As x increases, y only increases, so the range is $y \geq 30$.
4. Name what's excluded: the algebraic line continues to the left of x = 0, where y would be less than 30, even below 0. That part ($x < 0$) is not part of the model, because you can't have joined a negative number of months ago and it would mean paying a negative amount.

The idea: Not every context has a ceiling, sometimes only the negative side of x is restricted, and the range restriction just follows from plugging in the smallest allowed x.

Practice: Domain and range in context

1. A plant is 4 inches tall and grows 2 inches per week: $y = 4 + 2x$, where x is weeks since planting and y is height in inches. Which restriction on x is required by the story?

- **A** There is no restriction; x can be any real number including negative
- **B** $x \leq 4$, because the plant starts at 4 inches
- **C** $x \geq 0$, because weeks since planting can't be negative
- **D** x must be an even number, since growth happens in 2-inch steps

2. A car starts a trip with 12 gallons of gas and uses 0.5 gallons per mile: $y = 12 - 0.5x$, where x is miles driven and y is gallons left. At what mile does the tank reach empty?

A $x = 6$ **B** $x = 0.5$ **C** $x = 12$ **D** $x = 24$

3. A pool holds 500 gallons and is being filled at 25 gallons per minute starting from empty: $y = 25x$, where x is minutes and y is gallons in the pool. What is the realistic domain?

A $x \geq 0$ with no upper limit **B** $0 \leq x \leq 20$ **C** $0 \leq x \leq 500$ **D** All real numbers

4. A vending machine holds a maximum of 60 sodas. It starts full and sells sodas at a rate of 4 per hour during a football game: $y = 60 - 4x$, where x is hours since the game started and y is sodas left. Which part of the algebraic line is NOT part of the real model?

- **A** The parts where $x < 0$ or where $y < 0$ (equivalently $x > 15$)
- **B** Only the part of the line where $y < 0$, after the sodas have all run out
- **C** The part of the line where $0 \leq x \leq 15$, during the game itself
- **D** Only the part of the line where $x < 0$, before the game has begun

5. A phone plan costs $y = 25 + 0.10x$ dollars, where x is the number of texts sent in a month and y is total monthly cost. What is the realistic domain?

- **A** All real numbers, since cost can be any amount
- **B** $x \leq 0.10$, because that's the cost per text
- **C** $x \geq 0$, and x must be a whole number, since you can't send a negative or fractional number of texts
- **D** $x \geq 25$, because the plan has a $25 base fee

6. A hot air balloon is at 300 feet and descending at 15 feet per minute: $y = 300 - 15x$, where x is minutes and y is height in feet. The balloon lands when $y = 0$. A student says the domain is 'all real numbers because x is just a variable.' What is wrong with this reasoning, and what should the domain be?

(A) The domain should be $x \geq 0$ only, with no upper limit at all, since the balloon could simply keep on flying after it lands and the equation would keep giving values for y all the way down, so only the negative minutes need to be thrown out of the domain and nothing else does

(B) The reasoning ignores that x and y stand for real, measurable quantities in this story; the domain should be $0 \leq x \leq 20$, since negative minutes are impossible and the balloon can't descend below the ground ($y = 0$ at $x = 20$)

(C) The domain should be $0 \leq x \leq 300$, since 300 is the starting value

(D) The reasoning is completely correct as it stands, since the equation $y = 300 - 15x$ itself has no restrictions built into it anywhere, and any real number can be substituted for x to get a real number for y, so the domain has to be all real numbers no matter what story the equation is meant to describe

MATH · UNIT 1 · LESSON 8

When two plans cost the same



Goal: The student solves a one-variable linear equation that arises from setting two linear expressions equal (e.g., two phone plans costing the same), and interprets the solution as the point where two stories agree, without being told in advance that this is what the algebra represents.

About 50 minutes

Teach it FOR THE PARENT

1. **Say:** Two pizza places. Place A charges $6 delivery plus $1 per topping. Place B charges $2 delivery plus $2 per topping. Right now, which place seems cheaper to you? Is there a number of toppings where they'd cost the exact same amount? Don't calculate yet, just tell me what you think is going on.
   Look for: A guess that acknowledges the two costs start different and change at different rates, so they might cross somewhere, even a rough guess like 'small orders favor B, big orders favor A' counts.

2. **Say:** Here's the idea: when two things change at a steady rate, there can be exactly one amount where they're worth the same. We write an expression for each story, then set the two expressions equal to each other. That equals sign doesn't mean 'solve the right side', it means 'these two amounts are the same value right now.' Solving the equation finds the input where the two stories agree.
   Look for: Nothing to collect yet, this sets up vocabulary the student will apply next.

3. **Say:** Here is how to work the ride-share problem: App A = 3 + 2m, App B = 2.5m. I set them equal: 3 + 2m = 2.5m. Tell me, before I solve anything, what does this equation mean in words, in terms of the two apps?
   Look for: Something like 'it's asking for the number of miles where both apps cost the same amount,' not 'it's asking to compute 2.5m.'

4. **Say:** Now I solve it the way you did equations last year: subtract 2m from both sides, then divide. I get m = 6. Before I tell you what that means, what do you think m = 6 tells us about the two apps?
   Look for: Recognizes m = 6 is the mile mark where the costs are equal, ideally also predicting both costs will come out to the same dollar amount when checked.

5. **Say:** Your turn, fresh problem: Gym A costs $15 to join plus $4 per visit. Gym B costs $35 to join plus $2 per visit. Write an expression for each gym's total cost in terms of visits, v. Then set them equal and solve for v. Last step: tell me in a full sentence what your answer for v actually means about the two gyms.
   Look for: Correct expressions 15 + 4v and 35 + 2v, correct equation 15 + 4v = 35 + 2v, solved to v = 10, and a sentence stating that at 10 visits both gyms cost the same amount (finish by checking both sides equal $55).

6. **Say:** You've now done the real move: turning 'is there a point where two stories agree' into an equation, and reading the solution back as that point of agreement, not just a number. The practice set below gives you new stories. Some won't tell you it's an 'equal cost' question at all, you'll have to notice that for yourself.
   Look for: Readiness to move to independent items without being re-cued that 'set them equal' is the method.

If they get stuck

- *The equals sign means 'now write down the answer' rather than 'these two things are the same value', so an equation like 3 + 4 = 7 feels fine but 7 = 3 + 4 looks 'backwards', and an equation with expressions on both sides (used to find where two plans cost the same) is confusing because there's nothing to 'work out' on one side.*
  In the equation-writing and intersection lessons, deliberately present equations in reflexive and operations-on-both-sides form from the first worked example (not just a-op-b=c), and have the student verbally restate what each side represents in the context before manipulating symbols.
- *In an expression like '3b', the letter b is a label for the object itself ('3 buns') rather than a variable standing for a number (the number of buns), so combining or comparing terms with letters gets treated like combining objects rather than combining quantities.*
  In the equation-writing and function-notation lessons, require the student to state the units of every variable and every term aloud or in writing before symbols are manipulated ('m is a number of months, so 15m is a number of DOLLARS, not months'), do this as a fixed step, not a one-time warning.
- *When translating a comparison sentence like 'there are six times as many minutes used by plan A as by plan B' into symbols, the student writes the equation in the order the words appear (6A = B) rather than matching quantities correctly (A = 6B).*
  Teach a check-with-a-number step as a fixed routine: pick a small value for one quantity, compute what the other should be from the sentence, then test both candidate equations against that pair before committing.

Read together FOR THE STUDENT

Imagine two phone plans. Plan A costs $20 plus $5 for every gig of data. Plan B costs $10 plus $10 for every gig. At first Plan B looks cheaper, but the $10 rate adds up fast. There's one exact amount of data where both plans cost the SAME dollar amount. Finding that amount means writing '20 + 5g = 10 + 10g' and solving for g. The answer isn't just a number, it's the exact spot where two different stories about cost line up and agree.

Going further: When two situations each grow at a steady (linear) rate, you can write an expression for each one in terms of the same variable. Setting those two expressions equal to each other, Expression A = Expression B, doesn't mean 'calculate an answer on the right side.' It means 'find the input value where these two quantities are equal.' Solving that equation gives you the input (like months or gigs) where the two stories meet. On a graph, this is exactly the point where the two lines cross.

Watch it work

Ride-share App A charges a flat $3 pickup fee plus $2 per mile. App B charges no pickup fee but $2.50 per mile. Is there a trip length where both apps cost exactly the same? If so, how many miles?

1. Write an expression for each app's total cost in terms of miles, m: App A = 3 + 2m, App B = 2.5m. Each expression turns a cost story into math: the flat fee is a number by itself, and the per-mile price gets multiplied by m.
2. Set the two expressions equal: 3 + 2m = 2.5m. This says 'find the m where these two costs match', it is not asking you to compute a right-hand side, both sides are still unknown until you solve.
3. Solve for m using Grade 8 equation moves: subtract 2m from both sides to get 3 = 0.5m, then divide both sides by 0.5 to get m = 6.
4. Check what m = 6 means: at 6 miles, App A costs 3 + 2(6) = 15 and App B costs 2.5(6) = 15. Both stories agree at exactly 6 miles, that's the point where the two cost lines would cross on a graph.

The idea: Setting two expressions equal and solving finds the single input where two separate real-world stories produce the same output, the algebra is the search for agreement, not a left-to-right calculation.

Two tanks are being filled. Tank A starts with 5 gallons and fills at 3 gallons per minute. Tank B starts with 20 gallons and fills at 1 gallon per minute. After how many minutes do the tanks hold the same amount of water?

1. Turn each story into an expression in terms of minutes, t: Tank A = 5 + 3t, Tank B = 20 + t.
2. Set them equal because we want the moment the two amounts agree: 5 + 3t = 20 + t. Notice there are variable terms on both sides, this only makes sense if you read '=' as 'these amounts are the same,' not as 'now compute the right side.'
3. Solve: subtract t from both sides to get 5 + 2t = 20, then subtract 5 to get 2t = 15, then divide by 2 to get t = 7.5.
4. Substitute back to confirm: Tank A = 5 + 3(7.5) = 27.5, Tank B = 20 + 7.5 = 27.5. At t = 7.5 minutes both tanks hold 27.5 gallons, the point of agreement between the two fill stories.

The idea: The variable terms can sit on either side of the equation; what matters is that both expressions describe the same real quantity at the moment they're set equal.

Practice: When two plans cost the same

1. Two texting plans: Plan A costs 2 + 0.05t dollars for t texts, Plan B costs 0.10t dollars for t texts. A student sets up the equation 2 + 0.05t = 0.10t to find where the plans cost the same. What does this equation mean?

(A) It means: find the number of texts where both plans cost the same amount.

(B) It means Plan A always costs 2 dollars more than Plan B.

(C) It means: compute 0.10t first, then put that answer on the left side.

(D) It means t stands for the label 'texts,' so 0.05t means '0.05 texts.'

2. A student solves 4 + 3v = 10 + v (visits, v, for two gym plans) like this: '3v = 10 + v, so 3v - v = 10, so 2v = 10, v = 5.' They forgot to move the 4. What mistake did they make, and does it change what v=5 would mean if it were correct?

(A) They dropped the constant term 4 from the left side when combining terms, which is a step error, but if the arithmetic had been done correctly, v would still represent 'the number of visits where the two gym costs are equal.'

(B) The mistake is that v can't appear on both sides of the equation.

(C) The mistake means that v = 5 now represents the number of visits for gym A only, and not for both gyms at once, because dropping the 4 from the left side removed gym A's flat fee from the comparison, so the answer only tells us something about that one plan and nothing at all about where the two plans are equal.

(D) They made no mistake; v = 5 is correct.

3. Two candle sizes: Candle A is 10 cm tall and burns down 0.5 cm per hour. Candle B is 4 cm tall and burns down 0.8 cm per hour (both burning DOWN, getting shorter). Which equation correctly finds the hour, h, when both candles are the same height?

(A) 10 + 0.5h = 4 + 0.8h (B) 10 - 0.5h = 4 - 0.8h (C) 10 - 4 = 0.5h - 0.8h

(D) 0.5h = 0.8h

4. 'A parking garage charges four times as much per hour as a nearby lot.' If the lot charges $c per hour and the garage charges $g per hour, which equation correctly represents this sentence?

(A) g = 4c (B) c = 4g (C) 4g = c (D) g + 4 = c

5. Movers Company A charges $50 plus $25 per hour. Movers Company B charges $80 plus $15 per hour. A student sets up 50 + 25h = 80 + 15h and solves to get h = 3. What does h = 3 mean here?

(A) After 3 hours, both moving companies would charge the same total price.

(B) Company A is always exactly $3 cheaper than Company B, no matter how many hours the move takes.

(C) It takes 3 hours to do the math problem.

(D) After 3 hours, Company A charges $3.

6. A student is asked to write and solve an equation for: 'Streaming Service A costs $8 a month plus $3 per movie rental. Streaming Service B costs $20 a month with no rental fee. Find when the two services cost the same.' They write 8 + 3r = 20 and solve, getting r ≈ 4. A classmate says this is wrong because there's no 'r' term on the right side. Who is correct?

(A) The original student is correct, Service B's cost doesn't depend on rentals, so its expression is just the constant 20, and the equation is still comparing two full cost expressions.

(B) The classmate is correct about this one: every equation comparing two linear plans must have the variable on both sides of the equals sign, so the original student needs to write Service B's cost as 20r instead of just 20 before the equation can be solved to find when the two services cost the same.

(C) Neither of them is correct here: you can't compare a flat-rate plan to a per-rental plan with one single equation, since one side would have a variable and the other side would not, so the only way to find when they cost the same is to make a table of values for each service and look for a match.

(D) The original student is correct in the end, but only because r should really represent dollars and not rentals in this equation, since the question is asking about when the two costs match, and the variable in an equation about cost always has to stand for the money rather than for the number of things bought.

7. A student sees the sentence: 'A large water tank holds three times as much water as a small tank.' They let s = the small tank's volume and L = the large tank's volume, and write 3L = s. They solve a related problem and get an answer, then say, 'The large tank holds less water than the small tank, which matches real tanks I've seen where the bigger-looking one sometimes holds less.' What is the actual problem with their reasoning?

(A) The problem is that s and L are the wrong variable names and should be swapped.

(B) Their equation is correct exactly as written, and their real-world reasoning about the tanks correctly explains an unusual but perfectly valid situation, since a large tank really can end up holding less water than a small one if the small one is filled all the way up while the large one is left mostly empty, and the equation 3L = s captures that.

(C) The real problem is that volume can't be compared using multiplication at all, only addition, so the sentence should have been turned into L = s + 3 instead, meaning the large tank holds three more units of water than the small one; their real-world reasoning about the tanks is beside the point, since the operation itself was wrong from the start.

(D) Their equation reverses the comparison, it should be L = 3s, so the large tank holds three times the small tank's volume, not one-third of it, and their real-world justification is patching over an equation error rather than checking it.

8. Two savings accounts: Account A starts with $200 and loses $8 per week (a withdrawal plan). Account B starts with $50 and gains $8 per week. A student sets up 200 - 8w = 50 + 8w and solves for w, getting w = 9.375. They say, 'That can't be a real answer for weeks, so the accounts never actually meet.' Are they right?

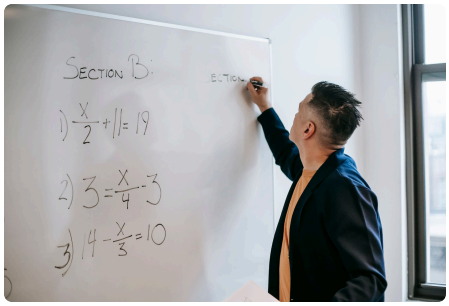
(A) They're right to dismiss it: since 9.375 isn't a whole number, the equation has no valid solution at all in this situation, and the two accounts never actually meet, because a savings balance can only be checked at the end of a whole week, and any answer that lands between two weeks is really telling us that the equation was not the right tool for the problem in the first place.

(B) They're wrong to dismiss it, a non-whole-number solution is still a valid point of agreement mathematically; it just falls between whole weeks, so if only whole-week balances are checked, the accounts pass each other without ever landing on the exact same value on a checked day.

(C) They're wrong to dismiss it, but only because the equation should have been set up as 200 + 8w = 50 - 8w in the first place, with the gains and losses switched between the two accounts; once that is fixed and solved the right way, w comes out to a whole number of weeks, and that whole number is the real point where the two accounts have exactly the same balance on a checked day.

(D) They're right, because w represents 'weeks' as a label, and labels can't be fractional.

Choosing when to use linear models



Goal: Given an entirely new kind of everyday situation not resembling any unit example (e.g., a recipe scaling with a fixed one-time setup cost analog), the student decides, without being told, whether a linear model is appropriate and produces all four representations, then argues in writing which representation someone with a specific decision to make should trust.

About 50 minutes

Teach it FOR THE PARENT

1. **Say:** Here's a new one: a 3D-printing shop charges a flat $15 setup fee to load your design file, then $0.50 for every gram of plastic used. Before I tell you anything else, is this going to be a straight-line relationship? How would you even check, without me telling you?
   Look for: Student proposes testing whether each extra gram adds the same fixed amount, not just assuming it's linear because it has a 'fee plus rate' shape.

2. **Say:** The whole game is this: linear isn't a shape you assume from a familiar-sounding setup, it's a claim you test. Pick two different jumps in the input and check if the output changes by the same amount both times. If yes, every representation you build afterward is trustworthy. If no, don't force a straight-line model onto it.
   Look for: Student can restate: constant rate of change across the relevant domain is the test for linearity, checked with real numbers, not assumed from context.

3. **Say:** Quick check before we move on: for the 3D-printing shop, going from 10 grams to 20 grams costs $5 more, and going from 20 grams to 30 grams also costs $5 more. Is it safe to call this linear? Why or why not, using the rate-of-change test?
   Look for: Yes, same rate ($5 per 10 grams, i.e. $0.50/gram) held across two different intervals, so the constant-rate test passed.

4. **Say:** Now you try a full one on your own, like Marta's garden example. A ride-share app charges a $2 booking fee plus $1.75 per mile, but only for trips under 10 miles. Trips of 10 miles or more get a flat $20 no matter the distance. Someone planning a 6-mile trip wants to know the exact fare before booking. Is the whole pricing scheme (0 to 15 miles) linear? For the piece that matters, type out a table, an equation, a story, and a description of the graph in words (you can also sketch the graph on paper if you have some, but you don't have to). Then tell me which of these the rider should trust.
   Look for: Student recognizes the relationship is piecewise (linear from 0-10 miles, flat/constant after 10, so NOT linear overall), builds correct table/equation/graph/story for the 0-10 mile linear piece only, and argues the equation (or exact evaluation) is most trustworthy for getting one precise fare, not the graph.

5. **Say:** Before we wrap up, if a friend told you 'any situation with a starting fee plus a rate per unit is always linear,' what would you tell them, using today's ride-share example as your evidence?
   Look for: Student pushes back: the fee-plus-rate shape only guarantees linearity if the rate stays constant across the whole range you care about; a cap, threshold, or change in rate breaks it, as shown by the ride-share flat-fee cutoff.

6. **Say:** You now have the real test: check the rate of change first, build all four representations only once you know it's linear, and pick the representation based on what the person actually needs to decide, exact number, trend, or meaning. The practice set has situations you haven't seen before. Use the test every time, don't pattern-match to garden fees or ride-shares.
   Look for: Student is ready to apply the constant-rate-of-change test to unfamiliar contexts and justify representation choice against a stated decision.

If they get stuck

- *The equals sign means 'now write down the answer' rather than 'these two things are the same value', so an equation like 3 + 4 = 7 feels fine but 7 = 3 + 4 looks 'backwards', and an equation with expressions on both sides (used to find where two plans cost the same) is confusing because there's nothing to 'work out' on one side.*
  In the equation-writing and intersection lessons, deliberately present equations in reflexive and operations-on-both-sides form from the first worked example (not just a-op-b=c), and have the student verbally restate what each side represents in the context before manipulating symbols.
- *In an expression like '3b', the letter b is a label for the object itself ('3 buns') rather than a variable standing for a number (the number of buns), so combining or comparing terms with letters gets treated like combining objects rather than combining quantities.*
  In the equation-writing and function-notation lessons, require the student to state the units of every variable and every term aloud or in writing before symbols are manipulated ('m is a number of months, so 15m is a number of DOLLARS, not months'), do this as a fixed step, not a one-time warning.
- *When translating a comparison sentence like 'there are six times as many minutes used by plan A as by plan B' into symbols, the student writes the equation in the order the words appear (6A = B) rather than matching quantities correctly (A = 6B).*
  Teach a check-with-a-number step as a fixed routine: pick a small value for one quantity, compute what the other should be from the sentence, then test both candidate equations against that pair before committing.

Read together FOR THE STUDENT

A linear relationship means: every time one thing goes up by the same amount, the other thing goes up (or down) by the same amount too, every single time, no exceptions. Think of a taxi that charges $3 just to get in, then $2 for every mile. Each mile always adds exactly $2. That steady, never-changing add-on is the sign of 'linear.' If the amount added changes, like a runner who gets more tired and slows down each mile, it's not linear. Before you build a table, graph, or equation, ask: does this situation add the same amount every single step? If not, a straight-line model will lie to you.

Going further: A situation is linear when it has a constant rate of change: equal steps in the input always produce equal steps in the output, giving one fixed slope everywhere. Many real situations only look linear over a limited domain, or have a rate that changes (fees that increase after a threshold, growth that compounds). Before producing a table, graph, equation, and verbal description, test the rate of change on at least two different intervals. If it's not constant, linear is the wrong model, say so and explain what breaks it.

When it is constant, each representation shows something different: the equation shows the rule for any input, the table shows exact values, the graph shows overall behavior and intersections, and the story explains what the numbers mean. Choosing which one to trust depends on what decision the person actually needs to make.

Watch it work

A community garden charges a one-time $25 fee to join, then $4 for every raised bed you rent for the season. Marta wants to know exactly how much she'd pay for any number of beds before she decides. Is this linear? If so, produce a table, graph, equation, and story description, then say which representation Marta should trust for her decision.

1. Check the rate of change first, before assuming anything: renting 1 bed costs 25+4=29, 2 beds costs 25+8=33, 3 beds costs 25+12=37. Each extra bed adds exactly $4, the rate is constant, so a linear model fits.
2. Build the equation from the two pieces of the story: the fixed $25 is the starting value (y-intercept), the $4 per bed is the constant rate (slope), so cost = 25 + 4b, where b = number of beds rented (b is a count, not a label, it multiplies by 4 to give dollars, not 'beds').
3. Make the table (b = 0,1,2,3 → cost = 25,29,33,37) and sketch the graph as a straight line starting at (0,25) with each step right-1 going up-4, this shows the trend but only approximates exact values between points if b must be a whole number.
4. Decide for Marta's actual decision: she wants an exact cost for a specific number of beds before committing money. The equation lets her plug in any number of beds and get an exact answer instantly, including numbers not shown in the table, so the equation is what she should trust, not the graph (which is for seeing trend/intersections) or the table (limited to listed values).

The idea: Before building any representation, test whether the rate of change is actually constant across the situation, linearity is a claim to verify, not a shape to assume from a familiar-sounding context.

A bakery says a wedding cake costs $60 for the base cake plus $8 per extra serving beyond the first 20 servings, but for the first 20 servings, the price is a flat $60 no matter how many of those 20 you use. A couple planning a 35-person wedding wants to compare this bakery to another one that charges a flat $12 per serving with no base fee, to decide which is cheaper for their guest count. Is the wedding-cake bakery's full pricing (from 0 to 35 servings) linear? Which representation should the couple trust to make their choice?

1. Test the rate of change across the whole domain the couple cares about (0 to 35 servings), not just part of it: from 0 to 20 servings, cost stays at $60 flat, that's a rate of $0 per serving. From 20 to 35 servings, cost rises by $8 per serving. Two different constant rates on two different intervals means the overall relationship from 0-35 is NOT linear, it's piecewise, even though each piece by itself is linear.
2. State what this means for representations: you cannot write one single linear equation for the whole 0-35 range; you'd need two equations, one per piece, or a graph showing a flat segment then a rising segment.
3. Compare to the second bakery, which IS linear the whole way: cost = 12s, where s = number of servings, constant rate $12 throughout, at 35 servings, that's $420.
4. For the couple's decision (which bakery is cheaper for exactly 35 servings), the graph is weakest here because reading an intersection precisely from two differently-shaped lines is error-prone; the table or direct equation evaluation at s=35 gives an exact, checkable number: bakery 1 costs 60 + 8(15) = $180, bakery 2 costs $420. The couple should trust the exact evaluated equation values (or equivalently a table at s=35), not a rough graph reading, and they should be told the first bakery's pricing isn't linear over the full range so a single slope claim about it would be wrong.

The idea: A relationship can be linear on each piece of its domain but not linear overall, always check the rate across the full range the decision actually needs, and match the representation to how precisely the decision requires the answer.

Practice: Choosing when to use linear models

1. A vending machine restocking service charges $10 to send a truck out, plus $0.75 for every bag of chips loaded. Going from 4 to 8 bags, cost rises by $3. Going from 8 to 12 bags, cost rises by $3. What does this tell you?

(A) The rate of change is constant across these intervals, so a linear model fits.

(B) Nothing yet, you'd need to see the equation first before checking if it's linear.

(C) It's linear only because the numbers are small and easy to work with.

(D) This can't be linear because the $10 truck fee is a one-time cost, not a per-bag cost.

2. For the vending machine service (cost = 10 + 0.75c, where c = number of bags of chips), what does the term 0.75c actually represent?

(A) The total number of bags of chips that are loaded into the truck for the service, not the money for them.

(B) 0.75 bags of chips for each trip, scaled up by the letter c to give the total number of bags loaded.

(C) The dollar amount added to the cost because of the chips loaded, not the chips themselves.

(D) The one-time setup cost of sending the truck out to the machine, which stays the same no matter what.

3. Two backyard chicken coops are compared: Coop Plan A costs 'four times as much per month as Coop Plan B.' Let A = Plan A's monthly cost and B = Plan B's monthly cost. Which equation correctly represents this?

(A) A + 4 = B (B) A = 4B (C) A = B + 4 (D) 4A = B

4. A moving company charges a flat $80 truck rental plus $30 per hour of labor, but only for jobs up to 6 hours. Any job of 6 hours or more is billed at a flat $300, no matter how much longer it actually takes. A customer estimates their move will take about 5 hours and wants to know the exact price before booking. Which representation should they trust, and why?

- **A** The graph, since it shows the overall trend of the pricing across all possible hours.
- **B** The story description, since it explains in words how the pricing works without needing any calculation.
- **C** The equation cost = 80 + 30h, applied to all possible job lengths including 6+ hours, since one linear equation should cover the whole pricing scheme.
- **D** The equation cost = 80 + 30h, evaluated at h = 5, since their estimate falls in the range where that equation applies and gives an exact number.

5. A streaming service's total cost is being modeled as cost = 12 + 9.99m, where m = number of months subscribed. A customer asks what cost when m = 3 actually means in the context. Which explanation is correct?

- **A** It means the total amount paid after subscribing for exactly 3 months, including the one-time setup charge.
- **B** It means 9.99 times 3 months, which equals 3 months of streaming time, so the answer is a number of months and not a number of dollars.
- **C** It means the setup fee is being charged 3 times.
- **D** It means the monthly rate charged during the 3rd month only.

6. A tutoring center says pricing is 'two times as much per hour for one-on-one sessions as for group sessions.' Let P = one-on-one price per hour, and let G = group price per hour. Which equation matches the sentence, and how would you check it?

- **A** P = G + 2, check by picking G = $15; then P = $17, which is $2 more per hour.
- **B** G = 2P: check by picking P = $15; then G = $30, so group costs twice as much as one-on-one, which is what the sentence says about the two prices.
- **C** P = 2G, check by picking G = $15; then one-on-one should be $30, which is twice as much, matching the sentence.
- **D** 2P = G: check by picking P = $15; then G = $30, meaning group costs more than one-on-one, which matches the sentence about two times as much.

7. A parking garage charges $5 just to enter, plus $3 for every hour parked. From 1 to 2 hours, cost rises by $3. From 2 to 3 hours, cost also rises by $3. Based on this, is the situation linear?

(A) Yes, equal hour-steps produce equal cost-steps, which is the test for a constant rate of change.

(B) No, the $5 entry fee being separate from the hourly rate means there are really two different rates.

(C) No, you can't call it linear until you've checked every single possible hour value, not just two intervals.

(D) Yes, but only because parking garages are always linear by nature.

8. For the parking garage (cost = 5 + 3h, h = hours parked), a customer says '3h means 3 hours.' What's the correct way to understand 3h in this equation?

(A) 3h is the dollar amount added to the cost because of the number of hours parked, h is a count of hours, and multiplying by 3 converts it to dollars.

(B) 3h means that the customer parked for exactly 3 hours in the garage, so the h just tells you that the 3 is a number of hours rather than a number of dollars, and the cost is then 5 plus those hours.

(C) 3h is the one-time entry fee.

(D) 3h literally means 3 hours of parking time.

MATH · UNIT 1 · LESSON 10

Slope and intercept vocabulary



Goal: The student recalls and correctly states the definitions of slope, x-intercept, and y-intercept when shown a labeled graph.

About 30 minutes

Teach it FOR THE PARENT

1. **Say:** Imagine a graph: a line crosses the up-down axis at (0, 2) and the flat axis at (4, 0). Before I tell you anything, which point do you think has a special name because x is 0 there?
   Look for: Student points to or names (0, 2) as related to the y-axis, showing they're already noticing the axis-crossing pattern.

2. **Say:** Here's the deal: the y-intercept is where the line crosses the y-axis, that's always the point (0, something). The x-intercept is where it crosses the x-axis, that's always (something, 0). Slope just tells you which way the line tilts and how steep it is: up-left-to-right is positive, down-left-to-right is negative.
   Look for: Nothing to check yet, this is direct explanation of the three terms.

3. **Say:** Quick check: on a graph, the line crosses the y-axis at (0, 5). Is that point the x-intercept or the y-intercept, and how do you know?
   Look for: Student says y-intercept, and justifies it with 'x = 0 there' or 'it's on the y-axis.'

4. **Say:** Let's label a full graph together. The line crosses (0, 4) and (-2, 0), and it rises left to right. Step 1: which axis does (0,4) touch? Step 2: which axis does (-2,0) touch? Step 3: which way does the line tilt?
   Look for: Student correctly identifies (0,4) as touching the y-axis (y-intercept), (-2,0) as touching the x-axis (x-intercept), and 'rises' as positive slope.

5. **Say:** Your turn, new graph: the line crosses (0, -3) and (6, 0), and it falls left to right. Name the y-intercept, the x-intercept, and the slope's direction.
   Look for: y-intercept = (0, -3), x-intercept = (6, 0), slope = negative, stated in those exact matched terms.

6. **Say:** You now know how to read three labels straight off a graph: where it crosses each axis, and which way it tilts. The practice set gives you graphs like these, read them the same way, point by point.
   Look for: Readiness to apply the same three-step read (y-axis crossing, x-axis crossing, tilt direction) to new labeled graphs in practice.

If they get stuck

- *The equals sign means 'now write down the answer' rather than 'these two things are the same value', so an equation like 3 + 4 = 7 feels fine but 7 = 3 + 4 looks 'backwards', and an equation with expressions on both sides (used to find where two plans cost the same) is confusing because there's nothing to 'work out' on one side.*
  In the equation-writing and intersection lessons, deliberately present equations in reflexive and operations-on-both-sides form from the first worked example (not just a-op-b=c), and have the student verbally restate what each side represents in the context before manipulating symbols.
- *In an expression like '3b', the letter b is a label for the object itself ('3 buns') rather than a variable standing for a number (the number of buns), so combining or comparing terms with letters gets treated like combining objects rather than combining quantities.*
  In the equation-writing and function-notation lessons, require the student to state the units of every variable and every term aloud or in writing before symbols are manipulated ('m is a number of months, so 15m is a number of DOLLARS, not months'), do this as a fixed step, not a one-time warning.
- *When translating a comparison sentence like 'there are six times as many minutes used by plan A as by plan B' into symbols, the student writes the equation in the order the words appear (6A = B) rather than matching quantities correctly (A = 6B).*
  Teach a check-with-a-number step as a fixed routine: pick a small value for one quantity, compute what the other should be from the sentence, then test both candidate equations against that pair before committing.

Read together FOR THE STUDENT

Imagine a graph with a straight line on it. Three words describe parts of that line. The y-intercept is the exact spot where the line crosses the up-down (vertical) line, called the y-axis. The x-intercept is the exact spot where the line crosses the flat (horizontal) line, called the x-axis. Slope is how steep the line is and which way it tilts. It tells you how much the line goes up or down as you move across. Think of slope like a ramp's steepness, and the intercepts like the two spots where a path crosses two roads.

Going further: On a graph of a linear relationship, the y-intercept is the point where the line crosses the y-axis, at that point, x = 0. The x-intercept is the point where the line crosses the x-axis, at that point, y = 0. Slope describes the steepness and direction of the line: how much y changes for each step across in x. A steeper line has a bigger slope number; a line going up left-to-right has positive slope, and one going down left-to-right has negative slope.

Watch it work

A graph shows a straight line. It crosses the vertical axis at the point (0, 4). It crosses the horizontal axis at the point (-2, 0). As you move from left to right, the line goes upward. State the slope's direction, the x-intercept, and the y-intercept.

1. Find where the line crosses the vertical (y) axis, that point is (0, 4), because the y-axis is where x = 0. This point is the y-intercept.
2. Find where the line crosses the horizontal (x) axis, that point is (-2, 0), because the x-axis is where y = 0. This point is the x-intercept.
3. Look at which way the line tilts as you scan left to right. It goes up, so the slope is positive.
4. State each part using its correct name: y-intercept = (0, 4), x-intercept = (-2, 0), slope direction = positive.

The idea: The y-intercept always has x = 0, and the x-intercept always has y = 0, you find each one by looking at which axis the line touches, not by calculating.

A graph shows a straight line crossing the horizontal axis at (3, 0) and the vertical axis at (0, -1). As you move from left to right, the line goes downward. State the x-intercept, the y-intercept, and the slope's direction.

1. Locate the crossing point on the horizontal (x) axis: (3, 0). Because y = 0 there, this is the x-intercept.
2. Locate the crossing point on the vertical (y) axis: (0, -1). Because x = 0 there, this is the y-intercept.
3. Check the tilt of the line from left to right, it goes down, so the slope is negative.
4. Report the three labels together: x-intercept = (3, 0), y-intercept = (0, -1), slope direction = negative.

The idea: Naming each feature only requires matching what you see on the graph to its definition, no calculation is needed when the graph is already labeled.

Practice: Slope and intercept vocabulary

1. A graph shows a straight line crossing the y-axis at the point (0, 6). What is this point called?

(A) The origin (B) The y-intercept (C) The slope (D) The x-intercept

2. A graph shows a straight line crossing the x-axis at the point (5, 0). What is this point called?

(A) The origin (B) The y-intercept (C) The slope (D) The x-intercept

3. A graph shows a line going upward as you move from left to right. What does this tell you about the slope?

(A) The slope is positive (B) There is not enough information to tell

(C) The slope is the y-intercept (D) The slope is negative

4. A graph shows a straight line crossing the y-axis at (0, -3) and the x-axis at (2, 0). Which statement correctly names both intercepts?

(A) The y-intercept is (0, -3) and the x-intercept is (2, 0)

(B) Both (0, -3) and (2, 0) are y-intercepts

(C) The x-intercept is (0, -3) and the y-intercept is (2, 0)

(D) The y-intercept is (0, -3) and the slope is (2, 0)

5. A graph shows a line falling as you move from left to right, crossing the y-axis at (0, 7). Which statement is correct?

(A) The y-intercept is (0, 7) and the slope is positive

(B) The y-intercept is (0, 7) and the slope is negative

(C) The y-intercept is 7 and the slope is (0, 7)

(D) The x-intercept is (0, 7) and the slope is negative

6. A student looks at a graph and says: 'The line crosses the x-axis at (0, -4), so that's the x-intercept.' What is wrong with this statement?

(A) Nothing is wrong; the statement is correct

(B) The point (0, -4) has x = 0, so it's actually on the y-axis, it should be called the y-intercept

(C) The slope of the line, and not the intercept, should be discussed here, since the point tells you how steep the line is

(D) The point should be written as (-4, 0) instead of (0, -4), since the x-intercept always has x first, but it is still on the x-axis

7. A graph shows a line crossing the x-axis at (-6, 0) and the y-axis at (0, 3), rising from left to right. A student writes: 'x-intercept = 3, y-intercept = -6, slope = positive.' Which part of this is incorrect?

(A) The intercepts should be written as single numbers and not as points, so both of the intercepts are wrong for that reason alone, and the slope is fine

(B) The x-intercept and y-intercept values are swapped, x-intercept should be (-6, 0) and y-intercept should be (0, 3)

(C) Nothing at all is incorrect in the student's statement: the x-intercept is 3, the y-intercept is -6, and the slope is positive, exactly as written

(D) The slope should be described as negative and not as positive, since the line crosses the x-axis at a negative number, but the intercepts are right

8. A graph shows a line that crosses the y-axis at (0, 0), the same point where it crosses the x-axis. What can you correctly say about this graph's intercepts?

(A) Only the y-intercept exists here, because the graph passes through zero on the y-axis first, and a line can only have one intercept at the origin

(B) This graph has no x-intercept and no y-intercept at all, since the two intercepts must always be different points and here they would be the same

(C) The x-intercept and y-intercept are both the point (0, 0), because that single point has both x = 0 and y = 0

(D) The slope of the line must be zero, because both of the intercepts are equal to each other, and equal intercepts always mean a flat line

Answer key

Math

Lesson 1	1. B 2. A 3. B 4. A 5. B 6. C 7. B
Lesson 2	1. A 2. A 3. C 4. A 5. D 6. B 7. D 8. B
Lesson 3	1. C 2. D 3. C 4. A 5. A 6. A 7. B
Lesson 4	1. C 2. D 3. A 4. C 5. C 6. D 7. C
Lesson 5	1. C 2. C 3. A 4. C 5. C 6. A 7. D 8. A
Lesson 6	1. C 2. A 3. C 4. B 5. A 6. B 7. D 8. D
Lesson 7	1. C 2. D 3. B 4. A 5. C 6. B
Lesson 8	1. A 2. A 3. B 4. A 5. A 6. A 7. D 8. B
Lesson 9	1. A 2. C 3. B 4. D 5. A 6. C 7. A 8. A
Lesson 10	1. B 2. D 3. A 4. A 5. B 6. B 7. B 8. C
Unit 1 check-up	1. C 2. D 3. B 4. C 5. D 6. B 7. D 8. D 9. B 10. C 11. B 12. C 13. C 14. A 15. C 16. D 17. C 18. A 19. C 20. D

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